Review of Trigonometric Identities

We've talked about trig integrals involving the sine and cosine functions. Now we'll look at trig functions like secant and tangent. Here's a quick review of their definitions:

$$\sec x = \frac{1}{\cos x} \qquad \tan x = \frac{\sin x}{\cos x} \tag{1}$$

$$\tag{2}$$

$$\csc x = \frac{1}{\sin x} \qquad \cot x = \frac{\cos x}{\sin x} \tag{3}$$

When you put a "co" in front of the name of the function, that exchanges the roles of sine and cosine in that function.

We have the following identities:

$$\begin{aligned} \sec^2 x &= 1 + \tan^2 x \\ \frac{d}{dx} \tan x &= \sec^2 x \\ \frac{d}{dx} \sec x &= \sec x \tan x \end{aligned}$$

We can verify these using familiar trig identities involving $\sin x$ and $\cos x$.

$$\begin{aligned} \sec^2 x &= \frac{1}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ \sec^2 x &= 1 + \tan^2 x \end{aligned}$$

This is the main trig identity behind what we'll do today.

$$\begin{aligned} \frac{d}{dx} \tan x &= \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) \\ &= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} \quad \text{(chain rule)} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ \frac{d}{dx} \tan x &= \sec^2 x \end{aligned}$$

From this we get our first integral of the day:

$$\int \sec^2 x \, dx = \tan x + c.$$

$$\begin{aligned}
\frac{d}{dx}\sec x &= \frac{d}{dx}\frac{1}{\cos x} \\
&= \frac{0-(-\sin x)}{\cos^2 x} \\
&= \frac{\sin x}{\cos^2 x} \\
\frac{d}{dx}\sec x &= \tan x \sec x
\end{aligned}$$

Should we ever need an antiderivative of $\tan x \sec x$ we now have one.