Parametric Curve

We're going to continue to work in three dimensional space, moving on to parametric equations; in particular we'll discuss parametric curves. This is another topic that will help us prepare for multivariable calculus; it's the beginning of the transition to multivariable thinking.

We're going to consider curves that are described by x being a function of t and y being a function of t.

$$\begin{aligned} x &= x(t) \\ y &= y(t). \end{aligned}$$

The variable t is called a *parameter*. The easiest way to think of parametric curves is as t equaling time and the position $(x(t), y(t))$ describing a *trajectory* in the plane.

The point $(x(0), y(0))$ describes a position at time $t = 0$. The point $(x(1), y(1))$ describes a later position at time $t = 1$. When we draw the trajectory it's a good idea to draw arrows on the curve that show what direction $(x(t), y(t))$ moves in as t increases.

Our first example will be to figure out what sort of curve is described by:

$$\begin{aligned} x &= a \cos t \\ y &= a \sin t. \end{aligned}$$

To do this we want to figure out what equation describes the curve in rectangular coordinates. Ideally, we quickly realize that:

$$\begin{aligned} x^2 + y^2 &= (x(t))^2 + (y(t))^2 \\ &= a^2 \cos^2 t + a^2 \sin^2 t \\ x^2 + y^2 &= a^2. \end{aligned}$$

The curve is a circle with radius a.

Another thing to keep track of is which direction we're going on this circle. There's more to this curve than just its shape; there's also where we are at what time, as with the trajectory of a planet. We'll figure this out by plotting a few points:

When $t = 0$, $(x, y) = (a \cos 0, a \sin 0) = (a, 0)$.

When $t = \frac{\pi}{2}$, $(x, y) = (a \cos \frac{\pi}{2}, a \sin \frac{\pi}{2}) = (0, a)$.

We deduce that the trajectory moves counterclockwise about the circle of radius a centered at the origin. (See Figure 1.)

Next time we'll learn to keep track of the arc length and to understand how fast $(x(t), y(t))$ is changing.

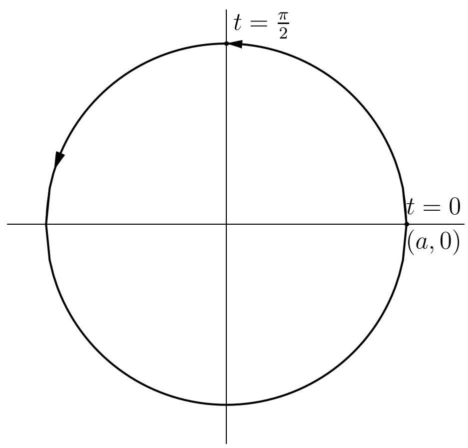

Figure 1: Parametrized circle.