Substitution of Power Series

We can find the power series of e^{-t^2} by starting with the power series for e^x and making the substitution $x = -t^2$.

$$
\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad (R = \infty) \\
e^{-t^2} &= 1 + (-t^2) + \frac{(-t^2)^2}{2!} + \frac{(-t^2)^3}{3!} + \cdots \\
&= 1 - t^2 + \frac{t^4}{2!} - \frac{t^6}{3!} + \cdots
\end{aligned}
$$

The signs of the terms alternate, the powers are all even, and the denominators are the factorials shown. The radius of convergence is infinity.

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