NAME

e-mail address(*)

code

(*) *By providing your e-mail address you are giving permission to your recitation instructor to e-mail you if did not pass the exam.*

Please mark the box next to your recitation session.

Recitation:	☐ Rec.#1	☐ Rec.#7
	☐ Rec.#2	☐ Rec.#8
	☐ Rec.#3	☐ Rec.#9
	☐ Rec.#4	☐ Rec.#10
	☐ Rec.#6	☐ Rec.#11

18.02 Exam 2 Thursday, Mar 16th, 2006

Directions: Do all the work on these pages; use reverse side if needed. Answers without accompanying reasoning may only receive partial credit. No books, notes, or calculators. Please stop when asked to and don't talk until your paper is handed in.

GRADING	
1.	/ 20
2.	/ 15
3.	/ 25
4.	/ 20
5.	/ 10
6.	/ 10
TOTAL	/ 100

Problem 1

Let $f(x, y) = xy^2 - 8y$.

a) *(5)* Find ∇f at $(2, 3)$.

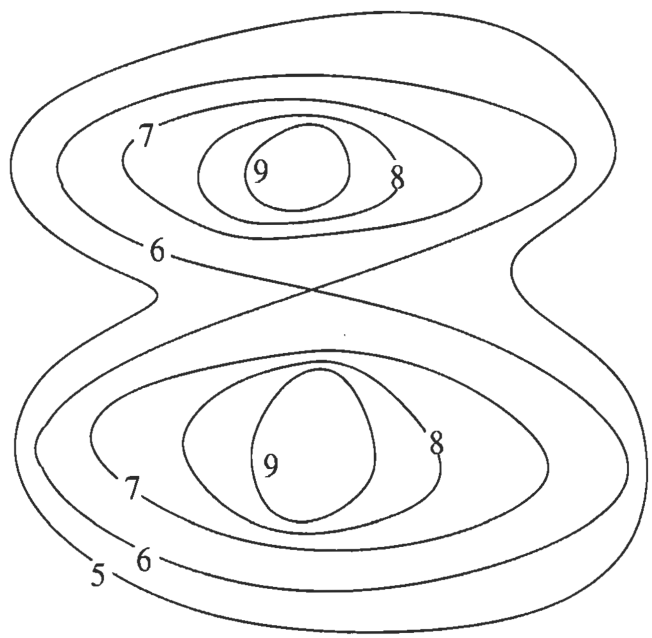
b) *(5)* Write the equation for the tangent plane to the graph of f through the point $(2, 3, -6)$.

c) *(5)* Use a linear approximation to approximate the value $f(2.1, 2.9)$.

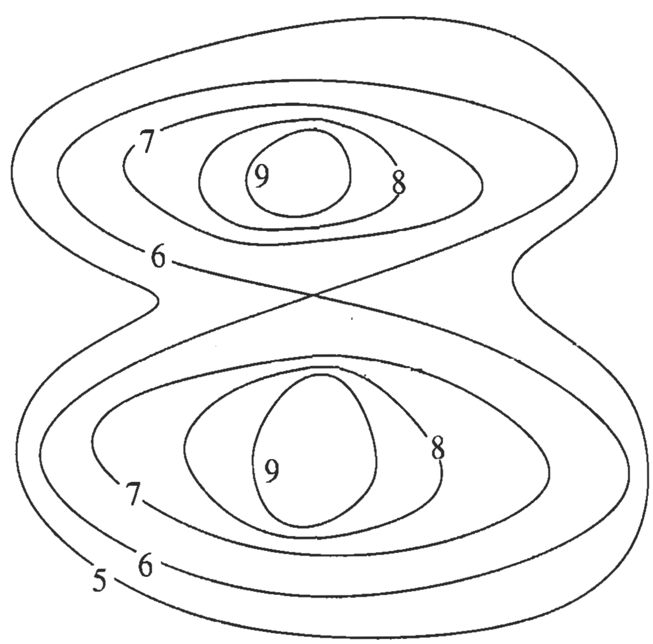
d) *(5)* Find the directional derivative of f at $(2, 3)$ in the direction $2\hat{\imath} + \hat{\jmath}$.

Problem 2

a) *(10)* On the contour plot below, mark the points of the level curve $f(x, y) = 6$ at which $f_x > 0$ and $f_y = 0$.



b). *(5)* On the contour plot below, mark the points of the level curve $f(x, y) = 8$ at which the slope of steepest ($|\nabla f|$ is largest).



Problem 3

Let $f(x, y) = x^2 + xy + y^2 + 3x$.

a) *(10)* Find and classify the critical points of f.

b) *(10)* Find the minimum and maximum values of f in the plane. Justify your answer.

c) *(5)* Find the minimum and maximum values of f in the region $x \geq 1$.

Problem 4

Suppose that $u = x + y^2$, $v = xy^{-2}$.

a) *(10)* Express the derivatives w_x and w_y in terms of w_u, w_v (and x and y).

b) *(10)* Express $xw_x + \frac{1}{2}yw_y$ in terms of w_u and w_v. Write the coefficients as functions of u and v.

Problem 5

(10) Set up (but do not solve) Lagrange multiplier equations for the point of the surface $2x^3 - yz^2 + xyz = 4$ closest to the origin.

Problem 6

Suppose that $f(x, y, z)$ is a function satisfying $\nabla f = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ at $(7, -8, 1)$ and that $z = z(x, y)$ is the root of the cubic equation $z^3 + xz + y = 0$. There is only one root z if $x > 0$ and, in particular, at $(x, y) = (7, -8)$, $z = 1$.

(10) Let $g(x, y) = f(x, y, z(x, y))$; find ∇g at $(x, y) = (7, -8)$.