Problem 1

(20) Find the mass of the solid cylinder $0 \leq x^2+y^2 \leq a^2, 0 \leq z \leq b$, with density $\delta(x, y, z) = x^2 z$.

Problem 2

(15) Express the average value of z^{10} on the surface of the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z > 0$, as an integral in spherical coordinates. **(Do not evaluate.)**

Problem 3

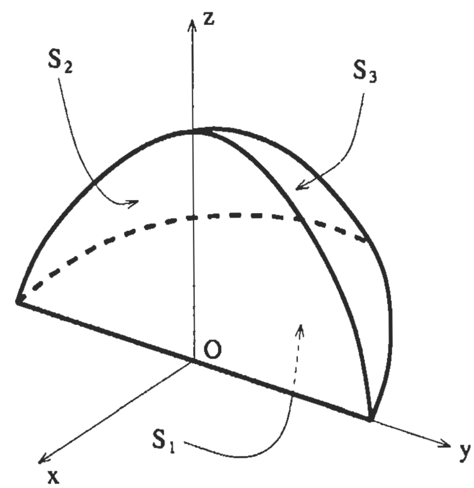
a) *(5)* Explain why $\boldsymbol{F} = \langle y, x + az, y + 1 \rangle$ cannot be a gradient field unless $a = 1$.

b) *(10)* Next, let $a = 1$. Then $\boldsymbol{F} = \langle y, x + z, y + 1 \rangle = \nabla(xy + yz + z)$.
Find $\displaystyle\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ for C given by $x = \cos^3 t$, $y = t$, $z = \sin^3 t$, $0 \le t \le \pi$.

Problem 4

Consider $\boldsymbol{F} = \hat{\imath}$ and D the solid quarter of a ball given by $x^2 + y^2 + z^2 < 1$, $x < 0$ and $z > 0$. Let $S = S_1 + S_2 + S_3$ denote the surface that encloses D, with S_1 the flat face in the xy-plane, S_2 the flat face in the yz-plane, and S_3 the curved face.

a) *(15)* State the divergence theorem, and use it to find the flux out of the curved face from the fluxes through the flat faces.



b) *(10)* Find the integrand $f(x, y)$ in the integral formula for the flux you found indirectly in part (a), that is,

$$\text{flux of } \boldsymbol{F} \text{ out of } S_3 = \iint_{x^2+y^2<1,\; x<0} f(x,y)\,dx\,dy$$

Do not evaluate the integral, and do not calculate the limits of integration. (The region of integration is the projection (shadow) of S_3 in the xy-plane.)

Problem 5

(25) Consider the surface S which is the portion of the plane $2y + z = 0$ in the cylinder $x^2 + y^2 \leq 1$. Its boundary curve C is the ellipse given by $x^2 + y^2 = 1$, $z = -2y$.
State Stokes' theorem, and confirm it by direct computation for $\boldsymbol{F} = z\hat{\imath}$ on S.

Problem 6 – *Extra credit (10 points)*

(10) Let $\boldsymbol{F} = y\hat{\imath} + 2z\hat{\jmath}$. Suppose that $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r} = 0$ for every curve in the plane $ax + by + cz = d$. What can be said about a, b, c, and d?