

## MODEL ANSWERS TO HWK #1

1.1.20. (a)  $\vec{0}$ . Suppose that  $\vec{a} = (a_1, a_2, a_3)$ . Then

$$0 \cdot \vec{a} = (0 \cdot a_1, 0 \cdot a_2, 0 \cdot a_3) = (0, 0, 0) = \vec{0}.$$

The case of a vector in  $\mathbb{R}^2$  is similar (and easier).

(b)  $\vec{a}$ . Suppose that  $\vec{a} = (a_1, a_2, a_3)$ . Then

$$1 \cdot \vec{a} = (1 \cdot a_1, 1 \cdot a_2, 1 \cdot a_3) = (a_1, a_2, a_3) = \vec{a}.$$

The case of a vector in  $\mathbb{R}^2$  is similar (and easier).

1.1.22. Let  $P_0 = (x_0, y_0, z_0)$  and let  $P = (x, y, z)$  be a general point of the parallelogram. Then

$$\overrightarrow{OP} = \overrightarrow{OP_0} + \overrightarrow{P_0P}.$$

Now

$$\overrightarrow{P_0P} = \lambda \vec{a} + \mu \vec{b},$$

where  $0 \leq \lambda \leq 1$  and  $0 \leq \mu \leq 1$ . (Indeed to get to  $P$  from  $P_0$ , one slides along the side parallel to  $\vec{a}$  and then slides in the direction of  $\vec{b}$ .)

So

$$\begin{aligned}(x, y, z) &= (x_0, y_0, z_0) + \lambda(a_1, a_2, a_3) + \mu(b_1, b_2, b_3) \\ &= (x_0 + \lambda a_1 + \mu b_1, y_0 + \lambda a_2 + \mu b_2, z_0 + \lambda a_3 + \mu b_3).\end{aligned}$$

1.1.24. (a) 4 mph.

(b) Since  $(5, 10) = 1/10(50, 100)$  it takes six minutes until the plane is directly above the skyscraper.

(c) In six minutes the plane climbs  $2/5$ th of a mile. Now one mile is 5,280 feet (google is your friend), so the plane is 2112 feet above the ground. So it clears the skyscraper by

$$2112 - 1250 = 862,$$

feet.

1.2.3.  $(3, \pi, -7) = 3\hat{i} + \pi\hat{j} - 7\hat{k}$ .

1.2.10  $\pi\hat{i} - \hat{j} = (\pi, -1, 0)$ .

1.2.11 (a) We want  $c_1$  and  $c_2$  such that

$$(3, 1) = (c_1 + c_2, c_1 - c_2),$$

that is, we want

$$c_1 + c_2 = 3$$

$$c_1 - c_2 = 1.$$

Adding both equations we get  $2c_1 = 4$ , so that  $c_1 = 2$  and subtracting both equations gives  $2c_2 = 2$ , so that  $c_2 = 1$ . It is easy to check that these values for  $c_1$  and  $c_2$  work.

(b) We want  $c_1$  and  $c_2$  such that

$$(3, -5) = (c_1 + c_2, c_1 - c_2),$$

that is, we want

$$\begin{aligned} c_1 + c_2 &= 3 \\ c_1 - c_2 &= -5. \end{aligned}$$

Adding both equations we get  $2c_1 = -2$ , so that  $c_1 = -1$  and subtracting both equations gives  $2c_2 = 8$ , so that  $c_2 = 4$ . It is easy to check that these values for  $c_1$  and  $c_2$  work.

(c) We want  $c_1$  and  $c_2$  such that

$$(b_1, b_2) = (c_1 + c_2, c_1 - c_2),$$

that is, we want

$$\begin{aligned} c_1 + c_2 &= b_1 \\ c_1 - c_2 &= b_2. \end{aligned}$$

Adding both equations we get  $2c_1 = b_1 + b_2$ , so that  $c_1 = (b_1 + b_2)/2$  and subtracting both equations gives  $2c_2 = b_1 - b_2$ , so that  $c_2 = (b_1 - b_2)/2$ . We check that these values actually work:

$$\frac{b_1 + b_2}{2}(1, 1) + \frac{b_1 - b_2}{2}(1, -1) = (b_1, b_2) = \vec{b},$$

as expected and required.

1.2.14.  $(x, y, z) = (12, -2, 0) + t(5, -12, 1) = (12 + 5t, -2 - 12t, t)$ .

1.2.16  $(x, y, z) = (2, 1, 2) + t(3 - 2, -1 - 1, 5 - 2) = (2 + t, 1 - 2t, 2 + 3t)$ .

1.2.24. If we plug in  $t = 0$ , then we see that the point  $(-5, 2, 1)$  lies on the first line. Now if this point lies on the second line, then we may find  $t$  such that  $(1 - 2t, 11 - 3t, 6t - 17) = (-5, 2, 1)$ . Comparing the first coordinates, we see that  $1 - 2t = -5$ , that is,  $t = 3$ . It is easy to see that then the second and third coordinates agree as well. So the two lines share the point  $(-5, 2, 1)$ .

If we plug in  $t = 0$  to the second line, then we get the point  $(1, 11, -17)$ . Now if this point lies on the first line, then we may find  $t$  such that  $(2t - 5, 3t + 2, 1 - 6t) = (1, 11, -17)$ . Looking at the first coordinate, we must have  $t = 3$ , and then it is easy to see that the second and third coordinates come out right.

So the two lines share two points. As any two points determine a unique line,  $l_1$  and  $l_2$  must indeed be the same line.

1.2.28. (a)  $(7, -2, 1)$  and  $(13, 1, -8)$ .

(b)  $(2, 1, -3)$ .

(c) When  $t - 2 = 1/6$ , so that  $t = 2 + 1/6$ , that is, after 130 seconds.

(d) The bird has  $y$ -coordinate equal to 4 after exactly six minutes. At this point its  $x$ -coordinate is 19, so no, the bird is never at this point (assuming the bird does not cheat and change directions on us).

1.2.30. We want to find  $t$  such that

$$5(1 - 4t) - 2(t - 3/2) + (2t + 1) = 1,$$

so that

$$-20t + 9 = 1,$$

and so  $t = -2/5$ . This is the point  $(1 - 8/5, -2/5 - 3/2, -4/5 + 1) = (-3/5, -21/10, -1/5)$ .

1.2.35. We want to know if we can find  $s$  and  $t$  such that

$$(2s + 1, -3s, s - 1) = (3t + 1, t + 5, 7 - t).$$

Adding the last two coordinates, we get  $-2s - 1 = 12$ , so that  $s = 13/2$ . Adding all of the coordinates together, we get  $0 = 3t + 13$ , so that  $t = -13/3$ . But then the third coordinate is a fraction with denominator 2, looking at the LHS and with denominator 3, looking at the RHS. As this is absurd, there are no such  $s$  and  $t$  and so the lines don't intersect.

1.2.38. Let  $P = (x, y)$ . The point  $A$  has coordinates  $(at, a)$  at time  $t$ . We have

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}.$$

Relative to  $A$ , the point  $P$  traces a circle, clockwise (anti, anti-clockwise, as it were), starting at the point  $(0, -b)$ . In other words the angle is  $3\pi/2 - t$ , at time  $t$  and so

$$\overrightarrow{AP} = (b \cos(3\pi/2 - t), b \sin(3\pi/2 - t)) = (-b \sin t, -b \cos t).$$

Therefore

$$(x, y) = \overrightarrow{OP} = (at - b \sin t, a - b \cos t).$$

There are machines, much like lawnmowers, whose job it is to make holes in the lawn. They have spikes instead of blades. If the point  $P$  is the endpoint of one of the spikes, then this is an example where  $b > a$ .

1.3.4.  $\vec{a} \cdot \vec{b} = 2 - 2 + 0 = 0$ ,  $\|\vec{a}\| = \sqrt{4 + 1} = \sqrt{5}$ , and  $\|\vec{b}\| = \sqrt{1 + 4 + 9} = \sqrt{14}$ .

1.3.8.  $\vec{a} \cdot \vec{b} = -1 + 2 - 2 = -1$ ,  $\|\vec{a}\| = \sqrt{3}$  and  $\|\vec{b}\| = 3$ . It follows that

$$-1 = 3\sqrt{3} \cos \theta \quad \text{so that} \quad \cos \theta = \frac{-1}{3\sqrt{3}}.$$

So  $\pi/2 < \theta < \pi$ . In fact

$$\theta \approx 1.764.$$

1.3.12.  $\vec{a} \cdot \vec{b} = 2 - 4 + 2 = 0$ . So  $\text{proj}_{\vec{a}} \vec{b} = \vec{0}$ .

1.3.13. Let  $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$ . Then

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} + \hat{k}),$$

is a unit vector which points in the direction of  $\vec{v}$ .

13.1.17. We suppose that neither  $\vec{a}$  nor  $\vec{b}$  is the zero vector.

$\text{proj}_{\vec{a}}\vec{b} = \text{proj}_{\vec{b}}\vec{a}$  if and only if either  $\vec{a}$  and  $\vec{b}$  are orthogonal or  $\vec{a} = \vec{b}$ .

If  $\vec{a}$  and  $\vec{b}$  are orthogonal, then both projections are the zero vector. If  $\vec{a} = \vec{b}$  then both projections are equal to  $\vec{a}$ . So one direction is clear.

Suppose that  $\text{proj}_{\vec{a}}\vec{b} = \text{proj}_{\vec{b}}\vec{a}$ . If  $\vec{a}$  and  $\vec{b}$  are not orthogonal, then both sides of this equation are non-zero vectors. As the LHS is parallel to  $\vec{a}$  and the RHS is parallel to  $\vec{b}$ , it follows that  $\vec{a}$  and  $\vec{b}$  are parallel.

In this case the LHS is equal to  $\vec{b}$  and the RHS is equal to  $\vec{a}$ . But then  $\vec{a} = \vec{b}$ .

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