Intersection of a line and a plane

1. Find the intersection of the line through the points $(1, 3, 0)$ and $(1, 2, 4)$ with the plane through the points $(0, 0, 0)$, $(1, 1, 0)$ and $(0, 1, 1)$.

Answer: This brings together a number of things we've learned. We must find the equations of the line and the plane and then find the intersection.

The basic data specifying a line are a point and a direction. We have

$$P_0 = (1, 3, 0) \quad \text{and} \quad \mathbf{v} = \langle 1, 2, 4 \rangle - \langle 1, 3, 0 \rangle = \langle 0, -1, 4 \rangle.$$

Therefore, the equations for the line are

$$x = 1, \quad y = 3 - t, \quad z = 4t.$$

The basic data specifying a plane are a point and a normal vector. We have

$$Q_0 = (0, 0, 0) \quad \text{and} \quad \mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \langle 1, -1, 1 \rangle.$$

Therefore the equation of the plane is

$$x - y + z = 0.$$

Substituting the line equations into the plane equation gives

$$1 - (3 - t) + 4t = 0 \iff t = 2/5.$$

We use this to find the point of intersection

$$(x, y, z) = (1, 13/5, 8/5).$$

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