CHRISTINE BREINER: Welcome back to recitation. In this video, I'd like us to do the following two problems, both related to the same position vector.

So we're starting off with a position vector defined as r of t is equal to 1 minus 2 t squared i plus t squared j plus negative 2 plus 2 t squared k. So that's our position vector, and I'd like us to do the following two things. And you'll notice this problem is pretty much just a computational problem. We're going to make sure that we know what these things I'm about to talk about are, how you define them, and how you get from the position vector to each of these things.

So we want to compute the velocity, the speed, the acceleration, and find the unit tangent vector for r of t. And then, the second part, we want to compute the arc length of the trajectory from t equals 0 to t equals 2. So I'll give you a moment to do that problem. Why don't you pause the video, work on the problem. When you're ready to check your work, bring the video back up and I'll show you how I do it.

OK, welcome back. Well, hopefully, you felt comfortable with answering these questions. So now I will answer them and you can compare your answers with mine.

So let me start off with part a. Part a, the first thing we're going to do is find the velocity. So velocity is really-- all we need to do is take the derivative of the position vector with respect to t. So I'm just going to take r prime of t. And now I'm going to write it in the shorthand notation that you've seen, with the brackets to denote that it's not a point, but it's a vector. So this is what you've seen to denote a vector rather than a point.

So the derivative with respect to t of the first component is just negative 4t. The derivative with respect to t of the second component is just 2t, because we had t squared, so when we take its derivative, we just get 2t. And the third component was negative 2 plus 2 t squared, so when I take its derivative, I get a 4t, so that is actually v of t, OK?

And then the next thing I asked you to do is determine the speed, and the speed, of course, is just the length of the velocity vector, right? So we just need to find the length of v. Now, to do that, to remind ourselves what we do for that, we actually take the inner product of v with itself, the dot product of v with itself, and then we take the square root of that. So let's look at what the dot product will be.

Let me find the squared thing first, and then I will take the square root. So v dotted with v, the first component I'm going to have negative 4t quantity squared, so that's going to be 16 t squared. And then the second component is going to be 2t quantity squared, so I'm going to have plus 4 t squared. And the third component is going to be another-- it's going to be 4t quantity squared, so I get another 16 t squared.

So when I add those up, I believe I get 36 t squared? Yes, good. And so then, I just have to take the square root of both sides to get what the speed actually is instead of the square of the speed. So I get 6t, OK? So that's the velocity; that's the speed. Now I need to find the acceleration and I need to find the unit tangent vector.

OK, so let me see. I will come over here. Let me step off here and I will find the acceleration and the unit tangent vector. So the acceleration, if you remember, the acceleration is actually just the derivative of the velocity with respect to t. So the acceleration is going to be the derivative of negative 4t is just negative 4. The derivative of 2t is just 2, and the derivative of 4t is just 4, all with respect to t. OK.

So the acceleration vector is equal to negative 4 comma 2 comma 4, so you see this actually has constant acceleration. So at any point, your acceleration is always this value, so it's not surprising that our velocity is increasing, and actually, it's increasing-- you'll notice, each of these components is constant. The velocity, each of the components is linear, and if we went back, we look at the position vector, each of those components is quadratic. And this is exactly what you expect from just your understanding of derivatives in single-variable calculus. If you start off with a constant and you find an antiderivative, it's going to be linear, and you find another antiderivative, you're going to have a quadratic, so we shouldn't be surprised by any of that.

Now we have one more thing to do with Part a, and that is to find the unit tangent vector. And that's fairly easy, because all we have to do is-- if you notice, we have the velocity vector and we have its length. And so to find the unit tangent vector, all we have to do is take the velocity and divide it by its length, and that will normalize it. That means that its length will be one at that point, because you're taking a vector, dividing by its length, so the length of the new vector will have to be length one.

So let me write that down. And actually, I guess the point to remember here is that the velocity vector is tangent to the path you're carving out, to the trajectory. OK, so this is a vector. This is a scalar. So I'm going to take 1 over 6t, and I'm going to multiply it by negative 4t, 2t, 4t, and this gives me, when I do my division, looks like it gives me a negative 2/3, right, 1/3, 2/3. So that is the unit tangent vector, OK?

OK, and now, we have one more point we want to make, and that is having to do with the arc length of the trajectory. That was the second part of this problem, was to find the arc length of the trajectory from t equals 0 to t equals 2. So let me just draw another line here. And what we want to do there then is-- really what we want to do is we want to integrate the speed, right? We want to integrate the speed from 0 to 2. So this-- let me come over here-- this absolute v, you might have also seen it written as ds/dt, right? And so we want to integrate this in dt-- in t, sorry-- from 0 to 2.

And so we come over here. We want to integrate from 0 to 2, 6t dt. That's fairly easy. That's going to be 6 t squared over 2, evaluated from 0 to 2. And so when I write that down, I'm going to get 6 times 4 divided by 2, 24 divided by 2, I just get 12, and the other term is 0. So the arc length is-- of the trajectory from 0 to 2 is just 12 units.

So this really was a purely computational type of problem. All we were doing, if you come back over here and you recall what we were trying to do, is we started off with a position vector. We just did a lot of computation. We found the velocity, the speed, the acceleration, the unit tangent vector, and then we just wanted to find the arc length of the trajectory. So this is all very computational, but just to make sure we understood what all the terms meant and how they were related to one another. So I'll stop there.