DAVID JORDAN: Hello, and welcome back to recitation. The problem I'd like to work with you now is simply to compute some partial derivatives, using the definitions we learned today in lecture.

So first we're going to compute the partial derivative in the x-direction of this function x y squared plus x squared y. Then we're going to compute its derivative in the y-direction, and then finally we're going to evaluate the partial derivative in the x-direction at a particular point (1, 2). That's the first problem. And in the second problem, we're going to compute second partial derivatives. Now these we just compute by taking the derivative of the derivative, just as we do in one-variable calculus. So why don't you work on these, pause the tape, and I'll check back in a moment, and we'll see how I solve these.

OK, welcome back. Let's get started. So we have x squared y-- excuse me, x y squared plus x squared y. That's our f. So when we take the partial derivative in the x-direction-- remember, this just means that we treat y as if it were a constant, and we just take an ordinary derivative in the x-direction as we would do in one-variable calculus. So the derivative of this in the x-direction is just y squared, because we only differentiate the x here. Similarly here, the derivative of x squared is 2x, and y just comes along for the ride as if it were a constant.

For the partial derivative in the y-direction, we do the same thing, except now, x is a constant, and we're taking an ordinary derivative in the y-direction. So we have 2xy plus x squared.

And then the final thing that we need to do is we want to evaluate partial f, partial x at the point (1, 2). And so all that means is that we have to plug in x equals 1 and y equals 2 into our previous computation, and so we get 2 squared plus 2 times 1 times 2. So altogether, we get 8. So that's computing partial derivatives.

Now let's move on and compute the second partial derivatives. So, for instance, we want to compute the second partial derivative both times in the x-direction. So all this means is that when we took the first partial, we got a function of x and y, and now we just need to take its partial. So we just need to take the derivative of this again in the x-direction. So now, the derivative of y squared-- be careful-- the derivative of y squared in the x-direction is just zero, because y is a constant relative to x. And so, then altogether, we just get 2y. When we take the derivative to this x, we just get 1. So that's our second partial derivative in the x-direction.

And now you can also take mixed partials. So here, we take a derivative of f. First we take the derivative in the y-direction and then we take a derivative of that in the x-direction. So we can look at our derivative here, partial f, partial y, and we need to take its partial in the x-direction. And so we get 2y plus 2x.

Now let's see what happens if we switch the order here and we take, instead, the partial derivative in the opposite order. So now let's go back to our partial derivative of f in the x-direction and let's take its derivative now in the y-direction. So the first term there, y squared, gives us a 2y and the second term gives us a 2x. I want to just note that these are equal.

In fact, the mixed partial derivatives, whether you take them in the xy order or the yx order, for the sorts of functions that we're going to be considering in this class-- for instance, all polynomial functions and all differentiable functions of several variables-- these mixed partials are going to be equal. In your textbook, there are some examples of sort of pathological functions where these are not equal, but certainly for any polynomial functions, these are always going to be equal. And I think I'll leave it at that.