## Problems: Gradient Fields and Potential Functions

Is the differential $y^{2} z d x+2 x y z d y+x y^{2} d z$ exact? If so, find a potential function. If not, explain why not.

Answer: The differential $f=M d x+N d y+P d z$ is exact if $M, N$, and $P$ are continuously differentiable in all of 3 -space and $P_{y}=N_{z}, M_{z}=P_{x}$ and $N_{x}=M_{y}$. Here $M=y^{2} z$, $N=2 x y z$ and $P=x y^{2}$ are continuously differentiable in all of 3 -space. We compute:

$$
\begin{array}{lc}
P_{y}=2 x y, & N_{z}=2 x y, \\
M_{z}=y^{2}, & P_{x}=y^{2}, \\
N_{x}=2 y z, & M_{y}=2 y z .
\end{array}
$$

This confirms that the differential is exact and so equal to $d f$ for some function $f(x, y, z)$. (Note that this method is equivalent to showing that $\operatorname{curl}\langle M, N, P\rangle=\mathbf{0}$.)
To find a potential function we could "guess and check" or integrate along a path like the one shown below. (We carefully chose the path to make the integrals as simple as possible.)

$$
\begin{aligned}
& \int_{0}^{x_{1}} 0 d x+\int_{0}^{y_{1}} 0 d y+\int_{0}^{z_{1}} x_{1} y_{1}^{2} d z=x_{1} y_{1}^{2} z_{1} .
\end{aligned}
$$

We conclude that the potential functions for this differential are of the form $f=x y^{2} z+C$.

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### 18.02SC Multivariable Calculus

Fall 2010

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