18.03 Recitation 1: February 2, 2010

Introduction, natural growth and decay, review of logarithm

An African government is trying to come up with good policy regarding the hunting of oryx. They are using the following model: the oryx population has a natural growth rate of k, and we suppose a constant harvesting rate of a oryxes per year.

Tasks:

1. Write down a model for the oryx population. [First step: choose symbols and units.]

2. Suppose $a = 0$. What is the *doubling time*?

3. Find the general solution of this equation.

4. Check that the proposed solution satisfies the ODE.

5. There is a "steady state" (also known as constant, or equilibrium) solution. Find it. Does the way the solution depends upon k and a make sense? (That is: do the units come out right? Does it behave right when a is large, or small, and when k is large, or small?)

6. Graph the steady state solution and some others, and solicit comments on what they signify. The equilibrium is "unstable." For initial values less than the equlibrium, the solutions stop having meanings in terms of this problem when they become negative. (Of course, this is true for all initial values, but for somewhat different reasons.) In that case, predict the time t_0 at which oryxes will be extirpated. For example, suppose that x_0 is half the steady state: what is t_0 (in terms of k)?

18.03 Differential Equations
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