18.034, Honors Differential Equations
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Lecture 14
3/5/04

1. Discussed linear differential operators:

$$L = a_n(t)D_n + a_{n-1}(t)D^{n-1} + \ldots\ldots + a_1(t)D + a_0(t)$$

   A. Bilinearity of L[y]
   B. Definition of composition of linear operators

$$(L_1 \bullet L_2)[y] = L_1[L_2[y]] \,.$$

   C. $L_1 \circ L_2 \neq L_2 \circ L_1$. But if L_1,L_2 are const. coeff. operators, $L_1 \circ L_2 = L_2 \circ L_1$,
   D. Discussed notation $p(D)$. Gives an association

   {Polynomials in 1-var} → {const. coeff. lin. diff. op's}.

   This map is compatible w/ addition + scaling & $(p_1 \bullet p_2)(D) = p_1(D) \bullet p_2(D)$. (so really an isom. of rings, but I didn't say this).

2. Used E.S.R. to "prove" that for the lin. diff. operator $p(D)$ w/ factor.
   $p(z) = (z\text{-}\lambda_1)^{l_1} + \ldots + (z\text{-}\lambda_1)^{l_0}$, gen'l sol'n is $y(t) = e^{\lambda_1 t}h_1(t) + \ldots. + e^{\lambda_0 t}h_0(t)$, $h_1 \bullet (t)$ a poly. of degree $\leq l_i - 1$. Compared # of params.

3. Discussed variant if p(z) has real coeffs.,

$$p(z) = (z - r_1)^{m_1} \ldots.(z - r_v)^{m_v}(z - \lambda_1)^{n_0}\left(z - \overline{\lambda}_1\right)^{n_1} \ldots (z - \lambda_w)^{n_w}\left(z - \overline{\lambda}_w\right)^{n_w} \,.$$

$$\begin{aligned} y &= e^{r_1 t}h_1(t) + \ldots + e^{r_v t}h_v(t) + e^{\alpha_1 t}\cos(\beta_1 t)g_1(t) + e^{\alpha_1 t}\sin(\beta_1 t)f_1(t) + \ldots \\ &\quad + e^{\alpha_v t}\cos(\beta_v t)g_v(t) + e^{\alpha_v t}\sin(\beta_v t)f_v(t) \end{aligned}$$