18.034, Honors Differential Equations
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Lecture 24
4/5/04

1. Quickly solved the IVP $\begin{cases} y'' + 2y + y = te^{2t} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$ by the

Laplace transform. Described the Heaviside coverup method and used it to compute $\overline{Y}(s)$.

2. Deduced that for a periodic function y(t) of period T, $\overline{Y}(s) = \frac{1}{e^{-sT}} \int_o^T e^{-sT} y(t)dt$.

For the periodic function, $y(t) = \begin{cases} +1, & 0 \le t < L, \\ -1, & L \le t < 2L \end{cases}$ $y(t+2L) = y(t)$,

deduced $\overline{Y}(s) = \frac{1}{s} \tanh\left(\frac{\delta L}{2}\right)$.

I made a point of the fact that the textbook's result about $p(D)y =$ periodic function f(t) in §5.3 can be derived directly w/o using the Laplace transform.

3. Proved that if $\overline{Y}(s)(s) = L[y, \overline{Z}]$, $\overline{Z}(s) = L[\overline{Z}(t)]$ and if

$w(t) := S(t)y * S(t)\overline{Z} = \int_o^T y(t-u)z(u)du$

then $\overline{W}(s) = L[w(t)]$ equals $\overline{Y}(s) = \overline{Z}(s)$.

Quickly explained significance to inhomog. linear const. coeff. ODE's, $p(0)y = f(t)$,
$y(0) = \ldots\ldots = y^{(n)}(0) = 0$.

Get $p(s)\overline{Y}(s) = F(s)$, $\overline{Y}(s) = (s) = \frac{1}{p(s)} F(s)$.

So if $k(t) = L^{-1}\left[\frac{1}{p(s)}\right]$, then $\boxed{y(t) = \int_o^T k(t-u)f(u)du}$