18.034 Honors Differential Equations
Spring 2009

18.034 Solutions to Problemset 3

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1. (b) $y_1'(0) = \frac{\omega}{\omega+\omega_0}\frac{1}{\omega-\omega_0} \to +\infty$ as $\omega_0 \to \omega$.

   (c) $y_2'(0) = -\frac{1}{\omega+\omega_0} \to -\frac{1}{2\omega}$.

   (d) $\lim_{\omega_0 \to \omega} y_2(t) = -\dfrac{1}{2\omega} t \cos \omega t$.

2. Birkhoff-Rota, pp. 28, Theorem 5.

3. (a) $c_1 \frac{\cos x}{x} + c_2 \frac{\sin x}{x}$

   (b) $c_1 x + c_2 e^{2x}$

4. (c) $\dfrac{1}{(1-x^2)^2} + \dfrac{n(n+1)}{1-x^2} \geq (n + y_2)^2$. Compare the solution with $\cos(n + y_2)x$.

5. Suppose $u(x) > 1$ at some point $a < x < b$. Then u takes a positive maximum > 1 at $a < c < b$. Observe that $u(c) > 1$, $u'(c) = 0$ and $u''(c) \leq 0$.

   At c, the differential equation reduces to

   $$\begin{array}{ccc} (\cosh c)u''(c) & = & (1+e^2)u(c) \\ \leq 0 & & > 0 \end{array}$$

   But this is a contradiction.

   The case $u(x) < 0$ at some point is completely analogous.

6. (a) $c_1 e^x + c_2 e^{-x} + c_3 e^{ix} + c_4 e^{-ix}$ or $c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

   (b) $c_1 e^{(1+i)x/\sqrt{2}} + c_2 e^{(1-i)x/\sqrt{2}} + c_3 e^{(-1+i)x/\sqrt{2}} + c_4 e^{(-1-i)x/\sqrt{2}}$ or $e^{x/\sqrt{2}}(c_1 \cos x/\sqrt{2} + c_2 \sin x/\sqrt{2}) + e^{-x/\sqrt{2}}(c_3 \cos x/\sqrt{2} + c_4 \sin x/\sqrt{2})$