MIT OpenCourseWare
http://ocw.mit.edu

# 18.034 Honors Differential Equations

Spring 2009

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

# 18.034 Recitation: April 7th, 2009

1. Volterra integral/Tautochrone example.

2. Suppose that $f$ and $g$ are piecewise continuous functions. Verify the following properties of their convolution.

   (a) $f * g = g * f$.

   (b) If $f \in C^1$, then $f * g$ is $C^1$ and $(f * g)' = f' * g$.

3. Use the Heaviside expansion to find an expression for the rest solution to the equation

$$y'' + 5y' + 6y = f(t).$$

   Verify your answer against the known solution in the case $f(t) = 1$.

4. *(Heaviside superposition formula)* Let $T$ be a linear differential operator with time-independent coefficients. Suppose that $f'$ is piecewise continuous, and $f$ continuous, and let $\phi$ be the rest solution to $T\phi = h(t)$ (here $h(t)$ denotes the unit step function). Express the rest solution to $Ty = f(t)$ in terms of $\phi$.

5. Consider the differential equation $y'' + y = h(t) - h(t - c)$ for $c > 0$.

   (a) Use the Laplace transform to find the rest solution.

   (b) Show that $y$ and $y'$ are continuous at $t = c$ but $y''$ is not.