Part I Problems and Solutions

Problem 1: *Change to polar form:*
a) $-1+i$
b) $\sqrt{3}-i$

Solution:

$$a)\ -1+i=\sqrt{2}e^{i3\pi/4} \qquad b)\ \sqrt{3}-i=2e^{-i\pi/6}$$

Problem 2: *Express* $\frac{1-i}{1+i}$ *in the form* $a+bi$ *via two methods: one using the Cartesian form throughout, and one changing numerator and denominator to polar form. Show the two answers agree.*

Solution: Using Cartesian form:

$$\frac{1-i}{1+i}=\frac{(1-i)^2}{(1+i)(1-i)}=\frac{-2i}{2}=-i$$

Other way:

$$\begin{aligned}1-i&=\sqrt{2}e^{-i\pi/4}\\ 1+i&=\sqrt{2}e^{i\pi/4}\\ \frac{1-i}{1+i}&=\frac{\sqrt{2}}{\sqrt{2}}\cdot e^{(-\pi/4-\pi/4)\,i}\\ &=e^{-i\pi/2}=-i\end{aligned}$$

Problem 3: *Calculate each of the following two ways: first by using the binomial theorem and second by changing to polar form and using DeMoivre's formula:*

a) $(1-i)^4$
b) $(1+i\sqrt{3})^3$

Solution:

$$\begin{aligned}a)\qquad (1-i)^4&=1+4(-i)+6(-i)^2+4(-i)^3+(-i)^4\\ &=1-6+1+i(-4+4)=-4\end{aligned}$$

By DeMoivre:

$$1 - i = \sqrt{2}e^{-i\pi/4}$$
$$(1-i)^4 = (\sqrt{2})^4 e^{-i\pi} = 4 \cdot (-1) = -4$$

$$b) \qquad \begin{aligned}(1+i\sqrt{3})^3 &= 1 + 3(i\sqrt{3}) + 3(i\sqrt{3})^2 + (i\sqrt{3})^3 \\ &= 1 + 3i\sqrt{3} + 3\cdot(-3) + i^3 3\sqrt{3} \\ &= -8 + i(3\sqrt{3} - 3\sqrt{3}) = -8\end{aligned}$$

By DeMoivre:

$$1 + i\sqrt{3} = 2e^{i\pi/3}$$
$$(1+i\sqrt{3})^3 = 8e^{i\pi} = -8$$

Problem 4: *Express the 6 sixth roots of 1 in the form $a + bi$.*

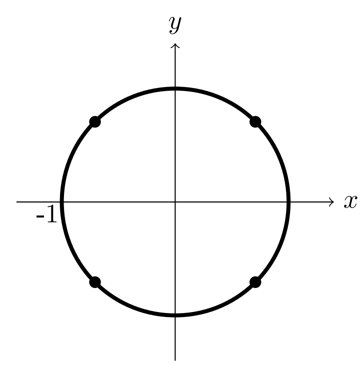
Solution: In polar form the sixth roots of 1 are $e^{i\pi k/3}$ where $k = 0, 1, 2, \cdots, 5$. Thus (using $\cos(\pi/3) = 1/2$, $\sin(\pi/3) = \sqrt{3}/2$) the roots in Cartesian form are

$$\pm 1, \quad \text{and} \quad \frac{\pm 1 \pm i\sqrt{3}}{2}$$

Problem 5: *Solve the equation $x^4 + 16 = 0$*

Solution: $\sqrt[4]{-16} = 2 \cdot \sqrt[4]{-1}$

The 4th roots of -1 are $e^{i(\pi/4 + n\pi/2} = \dfrac{\pm 1 \pm i}{\sqrt{2}}$. They are shown in the figure at right:



Thus, $\sqrt{2}\,(\pm 1 \pm i)$ are the roots of $x^4 + 16 = 0$.