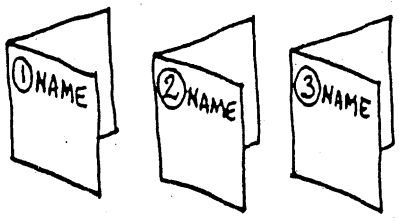
18.04 Exam #1 Friday, October 3, 2003

CLOSED BOOK ... and **NO calculators**

As before, please struggle with Problems 1, 2 and 3 on **separate** sheets of paper ...



(1) (a) Find the smallest positive integers m and n such that

$$(\sqrt{3} - i)^m = (1 + i)^n \ .$$

(b) Find all three solutions of $z^3 + (z+2)^3 = 0$.

(2) (a) Show that if a real function $F(x,y)$ is harmonic (= solves the 2D Laplace equation) then $F_x - iF_y$ is analytic.

(b) Show that if v is a harmonic conjugate of u then their product uv is also harmonic.

(3) (a) Somehow or other, verify the awesome identity

$$\tanh^{-1} z = \frac{1}{2} \log\left(\frac{1 + z}{1 - z}\right) \ .$$

(b) From this identity — even if you did not confirm it — deduce a tidy formula for

$$\frac{d}{dz} \tanh^{-1} z \ .$$