18.04 Handout   Monday, October 21, 1991

... of possible interest even in 2003

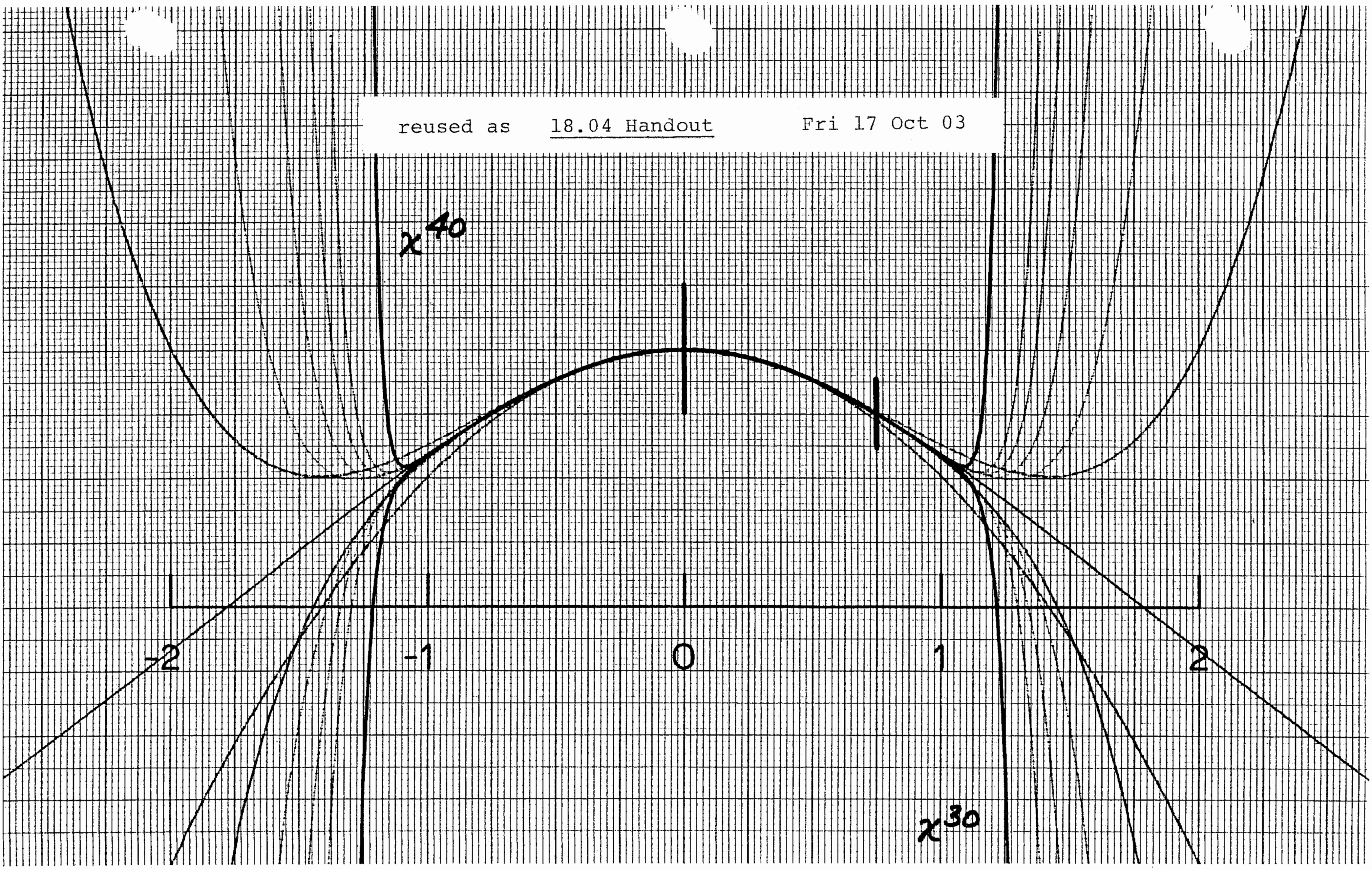
Here is how our ferocious $\oint \frac{dz}{(z^4 - 1)^2}$ could have been integrated **numerically** (!) once around the ellipse $(x+0.5)^2 + y^2/4 = 1$. FORTRAN program **PROB3.FOR** shown below produced the results boxed on the right.

```
      Program PROB3

      implicit double precision (a-h,o-z)

      pi = 4.0d0 * atan(1.0d0)
      answer = 3.0d0*pi / 8.0d0

      do 59 many=30,130,5

        a = 1.0d0
        b = 2.0d0

        xzero = -0.5d0

        sumR = 0.0d0
        sumI = 0.0d0

        dtheta = 2.0d0*pi/many

        do 29 k=1,many

          theta = k * dtheta

          ccc = cos(theta)
          sss = sin(theta)

          x = xzero + a*ccc
          y =           b*sss

          dx = -a * sss * dtheta
          dy =  b * ccc * dtheta

          z2R = x*x - y*y
          z2I = 2.0d0*x*y

          z4R = z2R*z2R - z2I*z2I - 1.0d0
          z4I = 2.0d0*z2R*z2I

          denR = z4R*z4R - z4I*z4I
          denI = 2.0d0*z4R*z4I

          den2 = denR*denR + denI*denI

          sumR = sumR + (dx*denR + dy*denI) / den2
          sumI = sumI + (dy*denR - dx*denI) / den2
   29   continue

        write (*,35) many, sumR, sumI, sumI-answer
   35   format (i10, f7.3, 2f14.9)

   59 continue

      end
```

many	Re ,	Im(gral)	Error
30	.000 ,	1.182121322	.004024077
35	.000	1.177733074	-.000364172
40	**.000**	**1.180819478**	**.002722233**
45	.000	1.177182065	-.000915180
50	.000	1.178725343	.000628098
55	.000	1.177891238	-.000206007
60	**.000**	**1.178197438**	**.000100193**
65	.000	1.178067378	-.000029867
70	.000	1.178109195	.000011950
75	.000	1.178094294	-.000002952
80	**.000**	**1.178098196**	**.000000951**
85	.000	1.178097130	-.000000115
90	.000	1.178097250	.000000004
95	.000	1.178097275	.000000029
100	**.000**	**1.178097229**	**-.000000016**
105	.000	1.178097254	.000000009
110	.000	1.178097241	-.000000004
115	.000	1.178097247	.000000002
120	**.000**	**1.178097245**	**-.000000001**
125	.000	1.178097245	.000000000
130	.000	1.178097245	.000000000

↑ $= \frac{3}{8}\pi$,

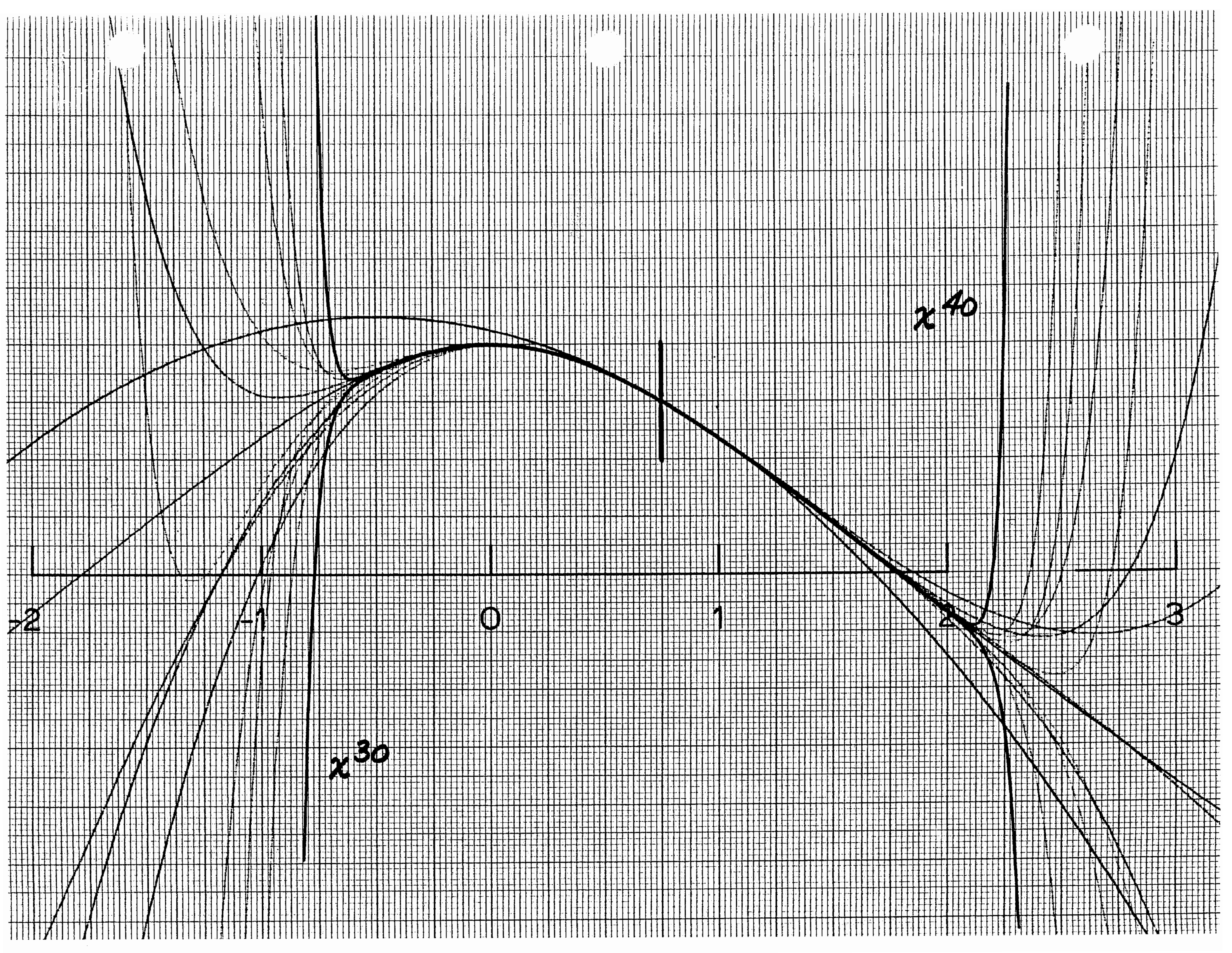
for some reason!



18.03 Briefly Revisited: Valiant attempts by Taylor Series to solve $(1 + x^2)\, y'' + y = 0$

The curves on this side illustrate not only the correct solution $y(x)$ satisfying $y(0) = 1$, $y'(0) = 0$, but also various Taylor polynomials that osculate it better and better near $x = 0$ ... and worse and worse for $|x| > 1$. The reverse diagram repeats this exercise starting from correct values at $x = 0.75$.

A.T.



x 40
x 30
-2
-1
0
1
2
3