18.04 Handout Wed 26 Nov 03

Back on **FOURIER** topics, here are some more old favorites ...

18.04 Problem Set 8   15 points   Due: Friday, Dec. 3, 1982

Yes, only a SAMPLE

1. In this first problem, let $f(\theta) = \dfrac{3}{5 + 4\cos\theta}$.

   (a) By residue calculus or (preferably) geometric series, figure out the surprisingly neat coefficients a_k needed for $f(\theta) = a_o + a_1 \cos\theta + a_2 \cos 2\theta + \ldots$

   (b) Use them to evaluate $\int_0^{\pi/2} f(\theta)\, d\theta$ to 4 decimals.

   (c) Use them also to show for $I = \int_0^{\pi} f(\theta)\, d\theta$ that the three-step midpoint estimate

   $$M_3 \equiv \frac{\pi}{3}[f(30°)+f(90°)+f(150°)] = 0.9692\ 30\ 769\,\pi$$

   differs from the accurate answer $I = \pi$ by a now very understandable amount.

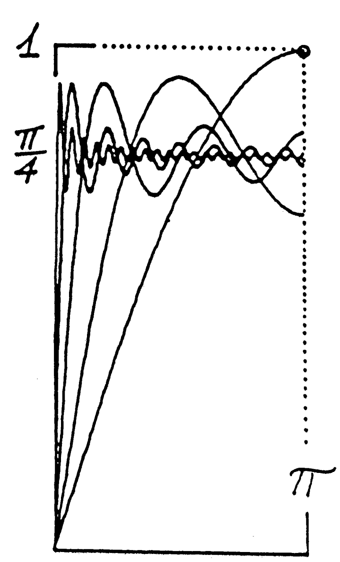
2. From the knowledge that $x = 2\,(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \ldots)$ for $|x| < \pi$, infer the classic sum

   $$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \frac{\pi^2}{6} .$$

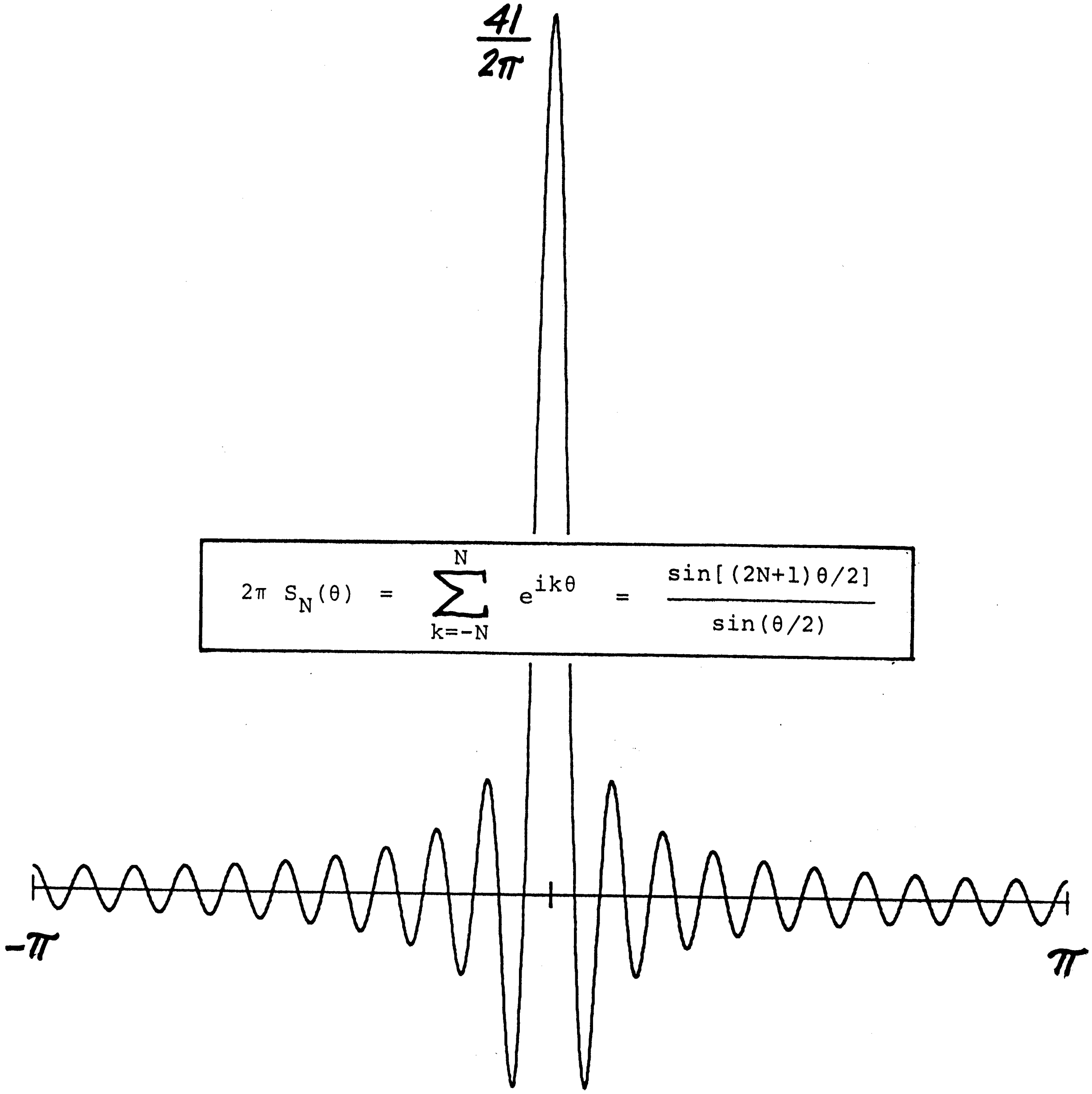
3. By courtesy of Mr. Gibbs, the truncated Fourier sine series (with k odd)

   $$S_k(x) = \sin x + \frac{1}{3}\sin 3x + \ldots + \frac{1}{k}\sin kx$$

   overshoots its intended $0 < x < \pi$ level $\pi/4$ in the manner plotted here for $k = 1, 3, 9, 27$ and 81 . Please calculate those overshoot heights H_k for $k = 1, 3$ and 9 . Also estimate to high accuracy $\lim_{k\to\infty} H_k$, most likely from some integral.



SCANNING FUNCTION $S_{20}(\theta)$



$$\boxed{2\pi\ S_N(\theta) = \sum_{k=-N}^{N} e^{ik\theta} = \frac{\sin[(2N+1)\theta/2]}{\sin(\theta/2)}}$$