

NIKOLA

Hi everyone. In this video, we're going to explore briefly the concept of a vector subspace.

KAMBUROV:

This is the problem we're going to do. We're given these four different subsets of \mathbb{R}^3 , and we are asked to figure out which of these is, in fact, a subspace. So I'll give you a few moments to try to do this on your own, and then please come back to see whether you were right.

So hi again. So before we start, let's briefly recall what a vector subspace is. Well vector subspace, of course, it's a subset. In this case, a subset of \mathbb{R}^3 . But it behaves like a vector space itself, meaning that if we take linear combination of elements in the subset, what we get is still something in that subset.

And in class, Professor Strang showed you a few specific examples of subspaces, which are related to matrices. One of them was the null space of a matrix, all the vectors that the matrix sends to 0, and the column space of a matrix, which is the span of the column of a matrix. So let's see if we can apply what we've learned in lecture to our problem.

Number one: we are given a linear equation relation between b_1 , b_2 , and b_3 . Once we have something linear-- this is a philosophy that you'll learn in this class-- we can always write this as a matrix equation.

So in particular, b_1 plus b_2 minus b_3 equals 0, we can just write it as the matrix $\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$, times $\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$ equal to 0. So therefore, b_1 , b_2 , and b_3 precisely describe the null space of $\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$. And thus, what we are given in the first question is a subspace.

What about the second one? Again, we are given the relation between b_1 , b_2 , b_3 . but it's not linear. b_3 is the product of b_1 and b_2 . So right, your guts tell you that this shouldn't be a vector space. But we need to prove why.

Well, see that the vector $[1, 1, 1]$ is inside this subset, because the third entry is the product-- 1 is the product 1 and 1. If the subset were a subspace itself, then we would have $[2, 2, 2]$ in it as well. Right, because any multiple of a vector in the subspace is inside the subspace. But is $[2, 2, 2]$ described by this equation? Well no, because the third entry, 2, is not equal to 2 times 2.

So the example in number 2 is not a subspace.

Let's try 3. And first-- so we are given-- so the subset in question 3 is given as the linear span of these two vectors, $[1, 0, -1]$ and $[1, 0, 1]$. Plus the vector $[1, 0, 0]$. So let's think about this geometrically. We know that the linear span of two linear independent vectors-- and these are obviously linearly independent-- is a plane in \mathbb{R}^3 . So we have the plane, and we add a vector to the point on this plane.

So what if the vector were lying on the plane? Well we're not going to change the plane. We're still going to remain in the plane. What I'm hinting at is the following thing. That, in fact, $[1, 0, 0]$ is a linear combination of $[1, 0, -1]$ and $[1, 0, 1]$, and it's fairly obvious to see which linear combination of these two vectors it is. It's $\frac{1}{2} [1, 0, -1]$ plus $\frac{1}{2}$ times $[1, 0, 1]$.

So we can write the whole relation here in the following way. $[b_1, b_2, b_3]$ is $[1, 0, 0]$, which we can write this, plus $c_1 [1, 0, -1]$, and $c_2 [1, 0, 1]$.

Let me continue this here. So $b_1, b_2,$ and b_3 is precisely c_1 plus $\frac{1}{2}$ of $[1, 0, -1]$ plus c_2 plus $\frac{1}{2}$ of $[1, 0, 1]$. So indeed, the points $b_1, b_2,$ and b_3 are described by the linear span $[1, 0, -1]$ and $[1, 0, 1]$. So it is a vector subspace itself.

And finally, let's look at number 4. So we have a similar situation. I mean, it's almost the same situation. We again have the linear span of precisely the same vectors as in question 3. But this time we add the vector $[0, 1, 0]$ to them.

Now, $[0, 1, 0]$ is not in the span of these two vectors. So the argument that we just showed for 3 doesn't work. But what we know about vector subspaces is the following thing. It follows almost trivially from the axioms that 0 needs to be inside the subset in order for it to be a subspace. It's a necessary condition.

Well is 0 inside this subset? Meaning can we find coefficients c_1 and c_2 , plus $[0, 1, 0]$ to equal zero vector? And the answer is no. And why is this? There's a very easy way to see it. Well we just look at the second entry of the vectors, and we see that any multiple-- so the linear span of these two vectors will have as its second entry 0 . And when we add it to 1 , we can never get a zero entry here. So the subset in 4 is not a subspace.

So I hope this was useful in just getting an intuition which subsets of \mathbb{R}^3 are subspaces. I'll see you guys later.