PROFESSOR: Hello, I'm Linan. Welcome to the recitation of Linear Algebra. It's my great pleasure to guide you through the first recitation.

In the first lecture, we learned some important concepts. We discussed how to view a linear system of equations from different points. And we discussed the concepts such as row picture, column picture, and the form in matrix. Some of them may be new to you. So today, we're going to use this simple example to review those concepts.

We're going to work with this simple system of two equations with two unknowns. So I would like you to first solve it, and then to find out the associated row picture and column picture with this system. After you're done, we're also going to discuss the matrix form of this linear system. Why don't you pause the video now and try to work them out on your own. A good suggestion would be, you can try to sketch your answer in an xy coordinate like this. OK, I'll see you in a while.

I hope you have just had some fun with it. Let's work on it together. Well, we're going to solve these equations. As you can see, we have two unknowns, x and y. And they have to satisfy these two equations at the same time.

How would you solve it? A very simple way would be you substitute x by y, in terms of y. So let's do it this way. So we use the second equation to rewrite x as 2y minus 1. So 2y minus 1. Then you plug this into the first equation.

This implies twice-- x would be replaced by this-- 2 times 2y minus 1 plus y is equal to 3. When you simplify this equation here, you will arrive at that 5y minus 2 is equal to 3. That simply tells you y is equal to 1. If y is equal to 1, then we go back to this formula. We see that x is also equal to 1.

That's it. This is the answer to this linear system. And it's easy enough. Both x and y are 1. Now, let's try to find out its row picture and column picture. So I'm going to work on this xy coordinate here.

First, let's look at row picture. Please review what a row picture is. So by row picture, I mean you have to look at this linear system according to each row. So what is the first row? Well, the first row is an equation with two unknowns.

So 2 times x plus y is equal to 3. What is this equation, exactly? As you may remember, this equation actually gives you a straight line in xy-plane. So now let's put the line here. I want a line that satisfies 2x plus y is 3.

Let's first set x to be 0. If x is 0, by the first equation, y should be 3, which is here. Now, let's set x to be 1. If x is 1, then y is also 1. So what I have now are two points on the line. And this is enough, because on the xy-plane, two points will uniquely determine a straight line. And that's the line we're looking for.

So all we need to do is to connect these two points. Let me try to draw this line straight. So this is the line that is given by 2x plus y is equal to 3.

This is the easiest way to determine a line. You just need to pick two points that are on the line. Then you connect them, and here you have the straight line. So that's the first row.

Now let's look at the second row. The second row is x minus 2y is equal to -1. Same thing. We're going to locate two points on the second line. So again, let's set x to be 0 first. If x is 0, then y should be 1/2, right?

So this line has to cross this point. That's x, 0; y, 1/2. Then let's put x to be 1 again. So if x is 1, then y is also 1. So we're going to use the same point here.

Now we have two points in the second line to connect them. So by connecting them, I will have my second straight line. So this is the line corresponding to x minus 2y is -1.

This is the row picture, because we have separated the two equations, and we look at them respectively. The first equation gives me this line, and the second equation gives me this line. Then what do I mean by solving this linear system? Well, we are putting these two lines together, and we are looking at a point which is on the first line and second line at the same time. Then clearly, that's the point where they intersect, right?

We can see from the answer over there, the coordinate of this point should be (1, 1). This is also clear from the construction of the two lines. We have noticed that this point is on the first line and the second line at the same time. Two lines meeting at the point (1, 1). That's the row picture of this linear system.

Now, let's move on to the column picture. Again, I will need an xy coordinate. So, where can I find my columns? If you look at the two equations, then you focus on the coefficient in front of

x in both equations.

What would that be? Well, in the first equation, I have a 2 in front of x. In the second equation, I have a 1 in front of x. I want to put them together as a column vector. Let me call it v_1.

And I'm going to do the same thing to the coefficients in front of y. In the first equation, I have a 1 in front of y. In the second equation, I have a -2 in front of y. Put them together. Call it a column vector v_2.

These are the columns I'm looking for. Then what does that linear system say? Well, I have extracted the coefficients in front of x. Now I can consider x to be the coefficient of this vector. And same thing, I'm going to view y as the coefficient of this vector. Then what that linear system is doing is just to sum them up.

That gives you the left-hand side of the linear system. What is the right-hand side? Again, you put the two constants as a column vector, which is [3, -1]. That is the right-hand side.

So what I'm doing here is I'm taking the linear combination of v_1 and v_2. And the coefficients are given by x and y, respectively. And I want the result of this combination to be [3, -1].

Now let's incorporate this into this picture. I have a v_1, so I'm going to draw a vector v_1. x is 2, y is 1, so v_1 is here. That's v_1.

And I need a v_2. x is 1. y is -2. So that's my v_2. I want to take the sum of x multiple of v_1 and y multiple of v_2. And I want the result to be [3, -1].

Well, taking a hint from the previous consideration, we know that both x and y should be 1. So I'm actually summing one copy of v_1 and one copy of v_2. So how do you indicate the sum of these two vectors? You complete the parallelogram spanned by these two vectors. Then the vector given by the diagonal is the sum of v_1 and v_2.

Is this vector going to be [3, -1]? Well, we can check. The x-coordinate will be 2 plus 1, which is 3. That's 2 plus 1, 3. And the y-coordinate will be 1 minus 2. So 1 minus 2, which should be -1.

That's it. That's one multiple of v_1 and one multiple of v_2. The sum will be [3, -1]. And that's the row picture. Where does that that "x is equal to y is equal to 1" come from? It comes from

solving the linear system. It comes from the row picture.

So here, we have found out the row picture and the column picture of this linear system. What I would like to mention is the matrix form of this linear system. So what is the matrix form? What if I put these two column vectors together? So I want to put them back to back, v_1 and v_2. And I call this matrix to be A. So if you write it out, A should be given by 2, 1; 1, -2. Matrix A has v_1 and v_2 as its column vectors, and it's a 2-by-2 matrix.

If I consider-- if I take this into account, then what will be the left-hand side of the linear system? In other words, what would be the left-hand side of this equation? This is actually matrix A multiplying a vector given by x, y. So that's [2, 1; 1, -2] multiplying x and y. So you put both unknowns together as a column vector. That's the left-hand side of the equation.

And again, the right-hand side is given by this column vector, [3; -1]. This is the matrix form of this linear system. We can actually solve this directly. In other words, we can get this unknown vector at once, both x-coordinate and y-coordinate.

Let's recall, if you have a scalar equation like this-- let's say a is some constant times x is unknown is equal to b. If a is non-zero, what would be x? So clearly, x should be b over a. I can also write it as a inverse times b. That's what we do when we have numbers.

So here, what we want to apply is a similar idea but to matrix. What you want to find is a matrix A inverse such that A inverse times A is equal to an identity matrix, which is [1, 0; 0, 1].

This may be new to you, but as you go further into this course, this idea will become more and more natural. If such an inverse matrix exists, then what would be this vector here? Then [x; y] will simply be A inverse times [3; -1]. That will give you the answer.

Here, I'm not going to go into detail, but we will return to this later in this course. I hope this simple example is helpful to you in reviewing what you've learned in the lecture. Thank you for watching, and I'm looking forward to seeing you again.