18.100A Fall 2012: Assignment 15

Directions You can collaborate, but should list collaborators, and write up the solutions independently, i.e., thinking through the problem by yourself and expressing it in your own words. Consulting solutions to problem sets of previous semesters is not allowed. Cite in the reasoning any significant theorems or definitions being used.

Reading: **18.1-.3; 18.4**(skip proofs). Riemann integrability: definition; integrabiity of continuous and monotone functions; properties of integrability,

Problem 1. (1) Read the second part of the proof of the Linearization Error Theorem (16.1B on p. 224) – i.e., the case $n = 1$ of Taylor's Theorem (17.2).
    Then do Question 16.1/3, if possible without consulting the answer.

Problem 2. (2) The Dirichlet function $\mathcal{D}(x)$ has the value 1 for x rational, and 0 for x irrational. Prove it is not Riemann-integrable.
    This is Question 18.2/3; do it if possible without looking at the answer.

Problem 3. (1) Work: 18.2/3.
    (This is easy, but requires careful attention to the definition of integrability.)

Problem 4. (3) Work: 18.3/1 , replacing n by k.
        (You'll need n for the n-partitions in your proof. Be careful–a single point can lie in more than one subinterval of a partition.)

Problem 5. (1) Work Question 18.3/3(a), if possible without looking at the answer.

Problem 6. (2) Work 18.3/4.
    (Given $\epsilon > 0$ and a large integer $N > 0$, give an upper estimate for $U(\mathcal{P})$ if $|\mathcal{P}| < \epsilon/N$.)

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