18.103 Fall 2013

Problem Set 2, updated version

[If you did not hand in AG §1.1: 18 and 19 with Problem Set 1, then do so with Problem Set 2.]

AG §1.1, pp. 11–14: 21.

AG §1.3, pp 39–42: 6, 10, 14, 17.

AG §1.4, pp. 49–52: 3, 5, 17.

Update: Here's the extra exercise that was promised. Show that if

$$\lim_{n\to\infty} X_n = 0$$

with probability 1, then for all $\epsilon > 0$,

$$\lim_{n\to\infty} \mathbf{P}(\{|X_n| > \epsilon\}) = 0$$

(For example, if $X_n = S_n/n$, $S_n = R_1 + \cdots + R_n$, then this says that the conclusion of the strong law of large numbers implies the conclusion of the weak law of large numbers. Hint: Use countable additivity.)

Hint for §1.3/10: Given a Cauchy sequence A_k, choose a subsequence $B_j = A_{k_j}$ with a geometric rate of convergence. Then let

$$A = \limsup B_j \equiv \bigcap_{\ell=1}^{\infty} \bigcup_{j\geq\ell} B_j$$

and show that A_k tends to A.

Remark on §1.4/17: Use a base 4 expansion to make a probabilistic model representing the random walk in the plane.