Homework 4; due Thursday, Oct. 17

1. Prove Mumford's theorem (see the notes, Th. 5.2) for $g = 1$.

2. Let $\Gamma(N)$ be the congruence subgroup of $SL_2(\mathbb{Z})$ which consists of matrices equal to 1 modulo N.

(a) Show that $\Gamma(N)$ is free for $N \geq 3$. (Hint: consider the action of $\Gamma(N)$ on the upper half plane).

(b) Find the number of generators of $\Gamma(N)$ which generate it without relations. (Hint: compute $\chi(\Gamma(N))$).

3. Let Γ be the group defined by the generators a, b, c with relations $ab = ba$. Find the Euler characteristic of Γ.