18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 07 2009 09 30 WED

TOPICS: Region of multiple values. Envelope of characteristics.

Continue with u_t + c(u)*u_x = 0 and u(x, 0) = F(x).
   Study boundary of the region of multiple values. Show that this
   is equivalent (as long as dc/du never vanishes) to looking at:
   c_t + c*c_x = 0, and c(x, 0) = C(x) = c(F(x)).
Relate boundary to maximums and minimums of x = z + c(z)*t for fixed t.
Write (parametric) equation for the curve.
Show curve is the envelope of the family of characteristics.
   Envelope of a (smooth) family of curves: locus of crossings of
   infinitesimally close members of the family. Find equations.
Behavior of the boundary produced by a local minimum (or maximum) of
   the initial data C(x).
   --- Local minimum: cusp pointing down-time in space time.
   --- Local maximum: cusp pointing up-time   in space time.