18.307: Integral Equations **M.I.T. Department of Mathematics**
Spring 2006

Problem Set 1 **Due:** Wednesday, 03/01/06

1. (Prob. 2.5 in text by M. Masujima.) Consider the integral equation

$$u(x) = 1 + \lambda \int_0^1 dy \, \frac{x^n - y^n}{x - y} \, u(y), \qquad 0 \le x \le 1.$$

(a) Solve this equation for $n = 2$. For what values of λ does the equation have no solutions?

(b) Discuss how you would solve this integral equation for arbitrary positive integer n.

2. (Prob. 2.4 in text by M. Masujima.) Solve the equation

$$u(\theta) = 1 + \lambda \int_0^{2\pi} d\phi \, \sin(\phi - \theta) \, u(\phi), \quad 0 \le \theta < 2\pi,$$

where $u(\theta)$ is periodic with period 2π. Does the kernel of this equation have any real eigenvalues?

3. (Prob. 8.17 in text by M. Masujima.) Solve the nonlinear equation

$$u(x) - \lambda \int_0^1 dy \, u^2(y) = 1.$$

In particular, identify the bifurcation points of this equation. What kind of point is $\lambda = 0$? What are the non-trivial solutions of the corresponding homogeneous equation?