Question and answers session.

Review envelopes and relationship between conservation laws and shocks.

Example 1. Energy conservation equation for shallow water:
Not correct to enforce it across hydraulic jumps.
C1 Conservation of mass/volume: $h_t + (h*u)_x = 0$,
C2 Conservation of momentum: $(h*u)_t + (h*u^2 + p)_x = 0$,
C3 Conservation of energy $E_t + (u*E + p*u)_x = 0$,

where $p = (1/2)*g*h^2$ is the average hydrostatic pressure,
and $E = (1/2)*h*u^2 + (1/2)*g*h^2$ is the energy density.

C1 and C2 are valid across hydraulic jumps, but C3 is not, since hydraulic jumps
dissipate. Note: conservation of momentum makes sense as long as we can neglect
the friction forces from the bottom. But, given this, there is no reason to expect extra
momentum dissipation at the shocks, which are rather thin.

Example 2. 2nd order models of traffic flow.
Derive Payne-Whitham, and explain traffic pressure in terms of preventive driving:
True velocity drivers aim at is $U-\mu*\rho\_x$, where $\mu = \mu(\rho) > 0$. [note $\nu = \mu*\rho$]

C1 Conservation of cars: $\rho_t + (\rho*u)_x = 0$,
E2 Proposed acceleration law: $u_t + u*u_x = (1/tau)*(U-\mu*\rho\_x-u)$,

where $U = U(\rho)$ desired velocity at equilibrium.

Equation E2 can be written in the form

E3 $u_t + u*u_x + (1/\rho)*p_x = (1/tau)*(U-u)$,

where $p=p(\rho)$ is defined by $p' = \mu*\rho/tau$. Here p is called the traffic pressure by
analogy with Gas Dynamics. Then E3 leads to the conservation of "momentum"
form:

C2 $(\rho*u)_t + (\rho*u^2 + p)_x = (\rho/tau)*(U-u)$.

However, momentum should not be conserved across traffic shocks, since cars
BRAKE there! Similarly, one can also write the following "conservation" laws [none
of which is likely to apply for true traffic shocks].

C3 $u_t + ((1/2)*u^2 + b(\rho))_x = (1/tau)*(U-u)$, where $b' = \mu/tau$.

C4 $E_t + (u*E + p*u)_x = (\rho*u/tau)*(U-u)$, where $E = (1/2)*\rho*u^2 + e(\rho)$, and $e = e(\rho)$ is
defined by: $(e/\rho)' = p/\rho^2$.

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