Gas Dynamics: Notice form $Y_t + A(Y)*Y_x$, similar to the scalar case, but with the wave velocity replaced by a matrix.
Look for o.d.e. forms [i.e. characteristics] by doing linear combinations of the equations. Need to find combinations that produce only one directional derivative in space-time.

HYPERBOLIC IN 1-D
Equations can be reduced to statements about directional derivatives of the solution.
Equivalent: A is real-diagonalizable.

Show it works if using eigenvalues/eigenvectors L*A = c*L:
Characteristic form: $L*(Y_t + c*Y_x) = 0$, or L*dY/dt = 0 along dx/dt = c.
Then, along the curves dx/dt = c, solution behaves (sort of) like an o.d.e.

Hyperbolic: have enough linearly independent (real) eigenvectors (in this case, 2) so that equation is equivalent to stuff above. This happens if and only if A is real diagonalizable.

Before applying these ideas to the full Gas Dynamics problem, LINEARIZE near the equilibrium solution u = 0, and $\rho = \rho_0$, and analyze the resulting problem (this is ACOUSTICS - NEXT ITEM/TOPIC below).

For a linear, constant coefficients, hyperbolic system: Y_t + A*Y_x = 0. Start with an orthonormal base of left and right eigenvectors:
Ln*A = cn*Ln
A*Rn = cn*Rn
Ln.Rm = delta_{n, m}
Then show general solution is Y = sum_n y_n(x-cn*t)*Rn.

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