

Show equivalent to wave equation:

Eliminate either R or u. Better yet, introduce velocity potential:

$$u = \phi_x \quad \text{and} \quad (a_0^2/\rho_0)R = -\phi_t$$

Note this is the pressure perturbation.

Hence, second equation is satisfied. Then first equation gives

$$\phi_{tt} - a_0^2 \phi_{xx} = 0. \quad \text{Wave equation.}$$

Note boundary conditions:

- Closed pipe: $\phi_x = 0$.
- Open pipe: $\phi = 0$.

Give other examples where these boundary conditions occur:

- Shallow water: Closed and open channel.
- String equation: Free end and clamped.

Using the solution above for gas-dynamics, we see that

$$\phi = F(x-a_0 t) + G(x+a_0 t),$$

where $F' = (1/2)f$ and $G' = (1/2)g$.

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