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Pak
9/26/05

$\Gamma_{n,k}$ = graph of k-colorings of $G_{n,n}$.

Then what's the diameter of Γ_{n3}?
(remember height function argument)
$\sum_{ij\in[n]^2} f(i_j) = O(n^3)$ (since each entry $O(n)$)
each decrease is size 2 $\Rightarrow$
$\mathrm{diam}(\Gamma_{n3}) = O(n^3)$

OTOH, $\mathrm{diam}(\Gamma_{nk}) = \Theta(n^2)$ for $k \geq 6$
and we see $\mathrm{diam}(\Gamma_{n4}) = D(n^3) = \mathrm{diam}(\Gamma_{n5})$

Theorem
G graph w/ m edges $\Rightarrow$
$\chi(G) \leq 1/2 + \sqrt{2m + 1/4}$
Pf: $m \geq \binom{k}{2}$ $k = \chi(G)$ (since in min'l proper coloring $\exists$ edge between any two colors

One More Obvious Theorem
$\chi(G) \geq n/\alpha(G)$
Pf: Duh.