Pak

10/5/05

$T_{C_3} = x^2 + x + y$, $T_{C_n} = x^{n-1} + T_{C_{n-1}}$

$\Rightarrow T_{C_n} = x^{n-1} + \dots + x + y$

Tutte polynomial

$$T_G(x,y) = \sum_{t \subseteq G} x^{\alpha(t)} y^{\beta(t)}$$

spanning tree $\quad \alpha(t), \beta(t)$ as before

want: Compute $T_G(x,y)$ where $G = K_n$

$= T_n(x,y)$ "let's make a notation"

$T_n(2,2) = 2^{\binom{n}{2}}$ $(2^{|E|})$ $\quad T_n(1,1) = \#\text{spanning trees} = n^{n-2}$

$T_n(1,2) = \#$ connected graphs on n vertices

$a_n = \#$ spanning trees in K_n $\quad (= T_n(1,1))$

$$a_{n+1} = \sum_{i=1}^{n} \binom{n-1}{i-1} i \, a_{n+1-i} \, a_i$$

split from root at 1 into comp't w/ max + rest, $\binom{n-1}{i-1}$ ways to partition vertices, i ways to connect two comp'ts

Check this gives $n^{n-2} = a_n$ by using generating f'ns

$$\sum_{i=1}^{n} \binom{n-1}{i-1} (n+1-i)^{n-1-i} \, i^{i-1} = (n-1)! \sum_{i=1}^{n} \frac{(n+1-i)^{n-1-i}}{(n-i)!} \frac{i^{i-1}}{(i-1)!}$$

Then look at $\sum_{n=0}^{\infty}$ " q^n, move on...

"n^{n-2} proofs of Cayley f'la"

$$T_{n+1}(x,y) = \sum_{i=1}^{n} \binom{n-1}{i-1} T_{n+1-i}(1,y)\, T_i(x,y)\, (x+y+\dots+y^{i-1})$$

Krewaras Gessel

Let $\angle$ be lexicographic order on $E(K_{nn})$
Note externally active in subtree iff e.a. in whole thing
$\Rightarrow$ both f'ns of y, internally active only in one comp't (since in other can substitute extreme vertex containing edge ξ) So split comp'ts by which has 2. Depending on which 1 neighbors in 2's comp't, # external + internal varies that's where you get $x+y+y^2+\dots+y^{i-1}$

Melov

If $C_n = T_n(1,2)$, then
$$C_{n+1} = \sum_{i=1} \binom{n-1}{i-1} C_{n+1-i}\, C_i\, (2^i - 1)$$