Pak
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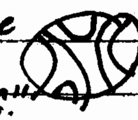
Grinberg's identity (EGT p 144)

Def'n Take a planar graph G w/ h.c. Put h. cycle around equator of sphere, edges as before.
Let f_i = # faces w/ i sides, f_i' = # in upper hemisphere
f_i'' = # in lower.

Thm (Grinberg) (1968)
$$\sum_{i\ge 3}(i-2)(f_i'-f_i'') = 0$$

Pf: $\sum_{i\ge 3} i f_i' = 2m' + n$ where m' = # edges in upper hem.

Note $\sum f_i' = m'+1$ since from above looks like 
so $\sum_{i\ge 3} i f_i' = n-2+2\sum f_i'$ and $\sum_{i\ge 3} i f_i'' = n-2+2\sum f_i''$

$\Rightarrow \sum_{i\ge 3}(i-2)f_i' = \sum_{i\ge 3}(i-2)f_i'' = n-2$ ✓

Theorem ~~Claim~~: Graph on p 145 of EGT is non-Hamiltonian
Pf: $f_9 = 1$ $f_8 = 3$ $f_5 = 21$
Suppose not. Then $3(f_5'-f_5'')+6(f_8'-f_8'')+7(f_9'-f_9'')=0$
$\Rightarrow f_9'-f_9'' \equiv_3 0$

Conj (Lovász)
Every connected vertex-transitive graph has a H path

Conj (Babai)
$\exists$ a sequence of conn. v-t graphs $\{G_n\}$ s.t.
$\ell(G_n) < (1-\varepsilon)|G_n|$ where $\ell(G)$ = length of longest path

Thm Suppose $\forall v \in V(G)$ $\deg v \geq K$. Then G (connected) contains a path of length $\geq 2K$. Moreover if $\deg v \geq n/2$ then G contains h c

Pf: Look at max'l path, $\ell < 2K \Rightarrow \exists$ edge w/ one vx neighboring on endpt + one neighboring other