Due in class: Fri 07 May 04

18.330 Problem Set 12

34 To great accuracy, here is a table of the **total resistances** R(N) for meshes similar to the one shown in Problem 30 for N = 4 , and studied there for N = 10 .

Your task now: Infer R(100) and R(1000) to comparable accuracy by **extrapolating** from just these data, using the handy asymptotic formula

N	Resistance
4	1.857142 857143
5	**2.136363** 636364
6	2.365656 565657
8	2.728976 763170
10	**3.011669** 564897
12	3.243022 584464
16	3.608519 738730
20	**3.892265** 540904
24	4.124203 027752

$$R(N) \sim (4/\pi) * \ln(N) + c_0 + c_2/N^2 + c_4/N^4 + c_6/N^6 + \ldots$$

35 Reemploy that 5 × 5 Pascal matrix

$$\underline{P} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{bmatrix}$$

to explore the prowess of the famous **Jacobi rotations** at inferring all five eigenvalues of this symmetric matrix.

To make your efforts easier to follow, tell us precisely for each of the first 10 rotations where and how big was the currently largest off-diagonal term which you thus chose to annihilate, and also report the sums of the squares of all 10 of the strictly upper (or lower) triangular elements which remained after each of those rotations.

Continue until you have diminished that sum of squares — which equalled 1604 to begin with — for the first time below 10^{-6} . How many such 2-D rotations did this take you?

36 To wrap up this term in heroic style, develop and turn loose some new 3-D **Gauss-Seidel** or **SOR** or **eigenfunction** scheme to confirm not only that the **total resistance**

(a) between corners A and B of the pictured cubic lattice consisting of 144 one-ohm resistors is 401/357 ohms, but also that

(b) this total will never reach even 1.5 ohms if the size of such a cube is made huge.