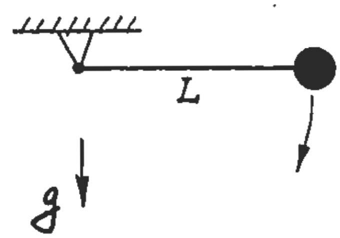
18.330 Problem Set 9 Due in class: Fri 16 Apr 04

(25) As a multiple of the period $2\pi\,(L/g)^{1/2}$ for oscillations of infinitesimal amplitude, figure out to our usual high accuracy how long it will take a simple pendulum of length L subject to gravity g to execute one complete cycle after starting from rest in the horizontal position sketched on the right. Work this out carefully as a **quadrature** problem based on the speed at each angle given by the obvious energy integral.



(26) Check this answer by using the **leapfrog** method (and maybe also RK4, for extra comfort but hardly any extra credit!) to advance

$$d\theta/dt = u\ , \qquad du/dt = -\sin\theta$$

onward by one-quarter of that period from the position shown.

PS: By leapfrog we here mean the scheme designed for $x'' = f(x)$ which begins from the known pair x_0, u_0 by first advancing the former variable to $x_1 = x_0 + hu_0$, and then hops onward like

$$u_{n+1} = u_{n-1} + 2h\ f(x_n)\ , \qquad x_{n+2} = x_n + 2h\ u_{n+1}$$

for $n = 1,3,5,\ldots,N-1$, finally terminating with another halfstep $x_N = x_{N-1} + hu_{N-1}$, where N is presumed to be even.

(27) One clever and efficient way of evaluating the **Bessel function** $J_0(x)$ and some kin like $J_1(x)$, $J_2(x)$, $J_3(x)$, ... starts from a deliberately "idiotic" pair of guesses like $J_{K+1}(x) = 0$ and $J_K(x) = 1$ and simply iterates backwards through the **recurrence relation**

$$J_{n-1}(x) + J_{n+1}(x) = (2n/x)\ J_n(x)$$

known to relate these functions at any given x . This trick is very similar to the "parasitic" solutions of certain multistep ODE methods like Milne or the 1-D leapfrog, but here that former nuisance is put to constructive use via the further identity

$$J_0(x) + 2J_2(x) + 2J_4(x) + 2J_6(x) + \ldots = 1$$

which permits the above mumbo-jumbo to be scaled back down to sane values afterward! Try this out for $x = 2, 4, 6, \ldots, 20$ and tell us what minimum values of the starting index K you found to be needed at each such x to obtain $J_0(x)$ correctly to 6 decimals.