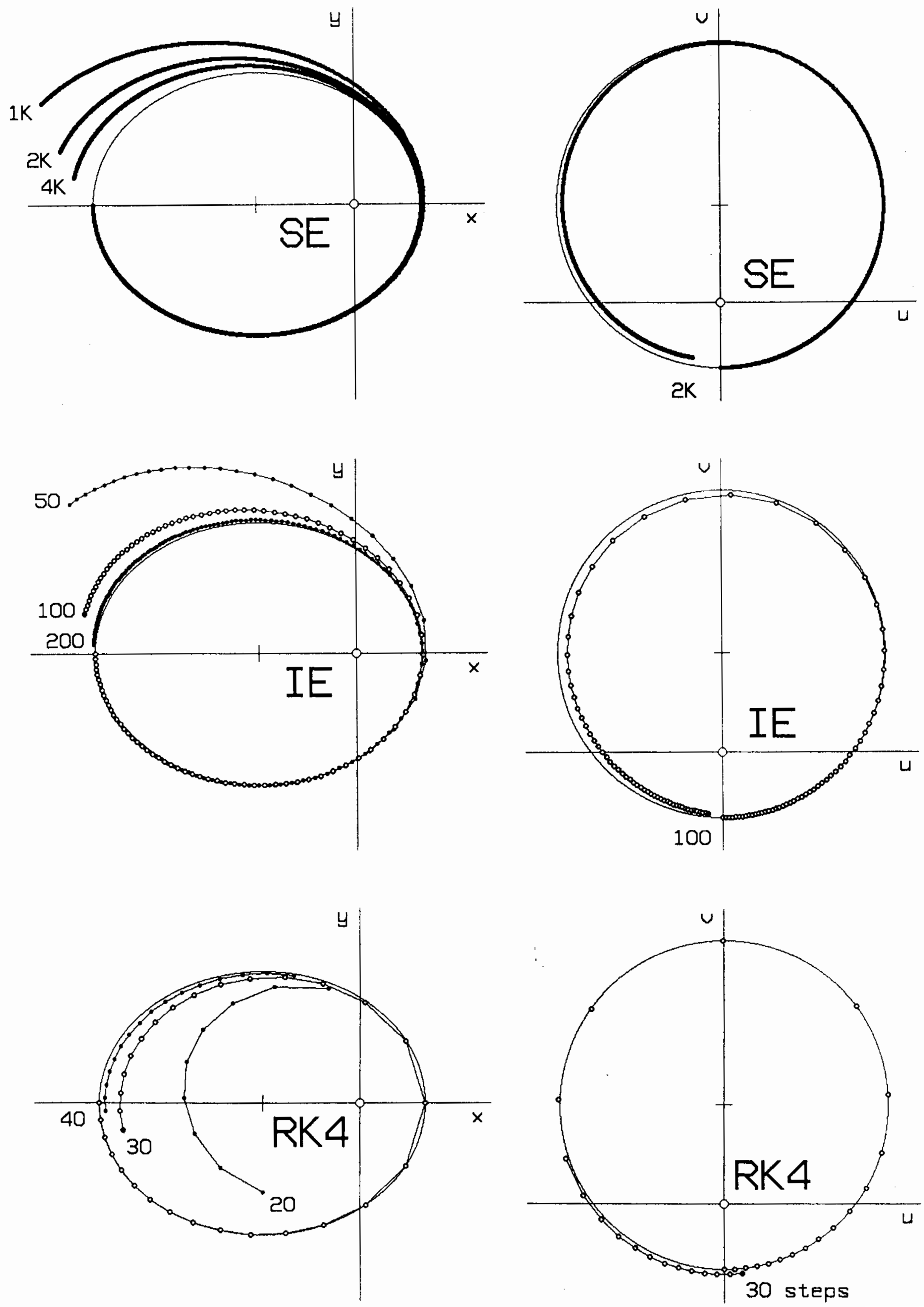
reused as **18.330 Handout** Wed 17 Mar 04

18.03 Handout Mon 05 Nov 01

ONE e = 0.6 KEPLER ORBIT ?!

according to **SE**, **IE** and **RK4**, when employing the indicated numbers of **equal** steps



PS: If you would like to check the performance of your **SE**, **IE** and/or **RK4** integrators for the **4-variable Kepler system**

$$dx/dt = u \qquad x(0) = -1.6$$
$$dy/dt = v \qquad \text{with} \qquad y(0) = 0$$
$$du/dt = -x / (x^2 + y^2)^{3/2} \qquad u(0) = 0$$
$$dv/dt = -y / (x^2 + y^2)^{3/2} \qquad v(0) = -0.5$$

illustrated in the front, here are 6-decimal versions of my own final values of x,y, u,v at $t = 2\pi$ obtained using each of the methods and (deliberately insufficient) numbers of steps N indicated:

	N	xfin	yfin	ufin	vfin
Simple Euler:					
	1000	-1.915385	0.615995	-0.352637	-0.333777
	2000	-1.801689	0.323027	-0.211429	-0.422452
	4000	-1.714675	0.164835	-0.117051	-0.464083
Improved Euler (= average of the end slopes):					
	50	-1.756273	0.914366	-0.368226	-0.276509
	100	-1.663363	0.238959	-0.110471	-0.467569
	200	-1.611030	0.052969	-0.022515	-0.496202
4th-order Runge-Kutta:					
	20	-0.600508	-0.541975	0.996085	-0.362168
	30	-1.449241	-0.163156	0.140100	-0.531470
	40	-1.562383	-0.050097	0.036481	-0.509762
versus		-1.6	0.0	0.0	-0.5

Of course those numbers do not yet match the initial quadruplet as they "ought" to have ... but bear in mind that all these calculations employed double precision (= approx. 14 significant digits) internally. Thus any big discrepancies here must have arisen from the step sizes dt having been chosen too coarsely, and **not** from round-off errors.

And yes, the **hodograph** (= locus in the u,v velocity space) for any Keplerian orbit should be strictly a **CIRCLE** !!

AT