

2.050J/12.006J/18.353J Nonlinear Dynamics I: Chaos, Fall 2012

Problem 1: 1D dynamical systems. Population growth. The Allee effect

Due at noon on Friday Sept 14th, in the box provided (to the right of the entrance).

No late pssets are accepted. If you collaborated with another student, list their names on the title sheet. The work that you submit must be your own.

Main concepts: phase space (phase line, phase plane), fixed points, their stability, review of linear algebra (matrices, eigenvectors, eigenvalues) and its application to linear systems of ODEs.

Reading: Strogatz Ch.2, 18.03 notes (LS.1-6) (see below)

Helpful review material from 18.03

- Review of 2nd order ODEs with constant coefficients – video lectures – see Lecture 9: <http://ocw.mit.edu/courses/mathematics/18-03-differential-equations-spring-2010/video-lectures/>
- 18.03 notes are on <http://math.mit.edu/supnotes/supnotes03/index.html>.

Problem 1: 1D dynamical systems. Population growth. The Allee effect

Strogatz 2.3.4

Problem 2: 1D dynamical systems. Potentials. Intro to bifurcations.

The dynamics of first order systems $\dot{x} = f(x)$, can be pictured as a particle sliding down the walls of a potential well, where the potential $V(x)$ is defined by $f(x) = -\frac{dV(x)}{dx}$ (Reference: Strogatz ch. 2.7).

1. Given the system $\dot{x} = \sin(x) + \alpha$, where α is a real number, find the potential $V(x)$, sketch it, show graphically how a particle would move for different values of α ($\alpha = 0, 0.9, 1, 1.1$).
2. For the same values α find fixed points, their stability, sketch a phase portrait (phase line) with arrows representing the flow between the fixed points, label the fixed points as stable, unstable, semistable.
3. Sketch a graph of the x -coordinate of the fixed points as a function of α , for $0 \leq \alpha \leq 1.2$. Describe what happens when α passes through 1? You just witnessed the phenomenon of a "bifurcation".

How would you define this concept based on the one example of a bifurcation that you just witnessed? (Do not look this concept up in the book, it has to be your definition that results from what you observe).

4. In another system $\dot{x} = (x + 1)(x - \alpha)$ with a parameter α , sketch the graphs of the x -coordinate of the fixed points, mark on the graph where the fixed point is stable or unstable
5. The phenomenon of a "bifurcation" occurs again at $\alpha = -1$. Describe what happens, and modify your definition of a "bifurcation" that you had above to include this case in the definition. Sketch the phase line for the values of α slightly below, at, and slightly above the bifurcation value $\alpha = 1$.
6. For the system $\dot{x} = (x + 1)(x - \alpha)$, sketch the phase line for the two values of α , before the bifurcation and after the bifurcation.

Problem 3: Linear oscillators (review of 18.03)

Given the equations for linear oscillators,

- find the general solution,
- classify the oscillators as underdamped, overdamped, critically damped or undamped,
- plot several trajectories as a function of time with different initial conditions.

- a) $\ddot{\theta} + 10\dot{\theta} + 41\theta = 0$,
- b) $\ddot{\theta} + \theta = 0$,
- c) $\ddot{\theta} + 11\dot{\theta} + 30\theta = 0$,
- d) $\ddot{\theta} + 6\dot{\theta} + 9\theta = 0$.

Problem 4: Linear systems of ODEs

For each of the equations in Problem 3,

1. change variables to (x, y) , where $x = \theta$, and $y = \dot{\theta}$,
2. rewrite the second order ODE as a system of first order ODEs for x and y and a matrix A ,
3. find eigenvalues and eigenvectors of the matrix A ,
4. find a general solution to the ODE,
5. (matlab) in the (x, y) plane plot some solutions (with different initial conditions) for each of the cases, plot eigenvectors (if they are real), and label the systems as underdamped, overdamped, critically damped or undamped. What can you say about the stability of the origin $(x, y) = (0, 0)$?

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