

2.050J/12.006J/18.353J Nonlinear Dynamics I: Chaos, Fall 2012

Problem Set 2

Due by the end of day on Thursday, in the box provided (until 5pm) ~~EN~~ No late psets are accepted. If you collaborated with other students in the class, list their names on the title sheet. The work that you submit must be your own.

Main concepts: phase portraits for linear systems, fixed points of nonlinear systems, their stability, phase portraits in the vicinity of a fixed point.

Reading: Strogatz Ch. 5, 6.1-6.4.

MATLAB help: there is a sample MATLAB code with examples of how to plot phase portraits and vector fields of a dynamical system on a plane.

Problem 1: Stability of fixed points of linear systems of ODEs

Strogatz 5.1.10

Problem 2: Romeo and Juliet. Linear systems of ODEs

If x is a measure of how attracted Juliet is to Romeo, and y is a measure of how attracted Romeo is to Juliet, and we model the evolution of their relationship as a linear system $\dot{x} = ax + by$, $\dot{y} = cx + dy$, then

1. do opposites attract? (i.e. analyze $\dot{x} = ax + by$, $\dot{y} = -bx - ay$).
2. is there a happy ending in the story of romantic clones $\dot{x} = ax + by$, $\dot{y} = bx + ay$?

Problem 3: Rabbits versus Sheep. Phase portrait for nonlinear systems of ODEs

The evolution of the populations of rabbits ($x \geq 0$) and sheep ($y \geq 0$) is described by $\dot{x} = x(3 - 2x - y)$, $\dot{y} = y(2 - x - y)$.

- Find fixed points, linearize the system near the fixed points, find eigenvalues, eigenvectors, determine the stability of the fixed point, and sketch the local phase portrait.
- Attempt to complete the sketch of the phase portrait for the full system
- (MATLAB) plot the phase portrait. Compare the plot to your sketch, note any differences.
- Describe different scenarios that the populations of rabbits and sheep would undergo depending on the populations at time zero.

Problem 4: Nonlinear systems which depend on a parameter (preview of bifurcations)

- Find fixed points, classify their stability (it will depend on α).
- Identify the values α^* of the parameter α , such that when α passes through α^* , the phase portrait qualitatively changes (fixed points appear, disappear, change stability, etc.). We say that at these values of α the "bifurcation" phenomenon occurs.
- Sketch (or plot with MATLAB) local phase portraits when α is just below, at, and just above α^* .
- As α passes through α^* , does the vector field far away from the fixed points change a lot?

a) $\dot{x} = \alpha x - x^2$, $\dot{y} = -y$,

b) $\dot{x} = \alpha x + x^3$, $\dot{y} = -y$.

Problem 5: Centers are not structurally stable

For the following nonlinear system $\dot{x} = -y + \alpha x^3$, $\dot{y} = x + \alpha y^3$ one of the fixed points is at the origin. Which phase portrait should the point have? It turns out that for systems with $Re(\lambda_i) = 0$ at the fixed point the linear analysis is inconclusive. We demonstrate it below.

a) linearize the system at the fixed point at the origin and confirm that linear analysis suggests that the fixed point is a center

b) change variables from (x, y) to polar coordinates (r, θ) using $x = r \cos \theta$, $y = r \sin \theta$, and the fact that $x^2 + y^2 = r^2$, and $\theta = \tan^{-1}(y/x)$

c) for this new system $\dot{r} = f(r, \theta)$, $\dot{\theta} = g(r, \theta)$, analyze the first equation (which you will probably have in the form $\dot{r} = f(r, x, y)$) and the fixed point $r = 0$. Show that for some values of α the origin $(x, y) = (0, 0)$ (which corresponds to $r = 0$) is stable, while for other values it is unstable. Comment on why this disagrees with the conclusion of the linear theory.

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