18.905 Problem Set 5

Due Wednesday, October 11 in class

---

1. Hatcher, Exercise 7 on page 155.

2. Hatcher, Exercise 9 on page 156.

3. Suppose $X = A_1 \cup A_2 \cup \cdots \cup A_n$, each A_i is open, and that every nonempty intersection $A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_r}$ is contractible. Show $H_k(X) = 0$ for $k \geq n-1, k \neq 0$.

   (This shows that a union of n open convex sets in $\mathbb{R}^m$ has homology concentrated below dimension $n-1$.)

4. Suppose that $\phi : E \to F$ is a map of generalized homology theories, as in the last problem set, such that for all n the map $\phi_n : E_n(pt, \emptyset) \to F_n(pt, \emptyset)$ is an isomorphism. Show that the map $\phi_n : E_n(X, A) \to F_n(X, A)$ is an isomorphism for any n and any finite CW-complex X with subcomplex A. (Hint: Prove it first for (D^n, S^{n-1}) and then apply induction.)