HOMEWORK 10

DUE: MONDAY, 5/1/06

1. (Hatcher's spectral sequence notes, Section 1.2, problem 1) Use the Serre spectral sequence to compute $H^*(F;\mathbb{Z})$ for F the homotopy fiber of a map $S^k \to S^k$ of degree n for $k, n > 1$, and show that the cup product structure in $H^*(F;\mathbb{Z})$ is trivial.

2. Inductively using the Serre spectral sequences for the fiber sequences

$$U(n-1) \to U(n) \to S^{2n-1},$$

show that there is an isomorphism

$$H^*(U(n)) \cong \Lambda[e_1, e_3, e_5, \dots, e_{2n-1}]$$

(an exterior algebra on generators in degrees $2i-1$).

3. Show that for complex line bundles L_i over a space X, $c_1(L_1 \otimes L_2) = c_1(L_1) + c_1(L_2)$. (Hint - consider the "universal example": the external tensor product $L_{univ} \otimes L_{univ}$ over $\mathbb{C}P^\infty \times \mathbb{C}P^\infty$).

4. (This problem requires Wed's lecture) Suppose that

$$\alpha : G \to G$$

is an inner automorphism (conjugation by an element of G). Show that the induced map

$$\alpha_* : BG \to BG$$

is homotopic to the identity.