MIT OpenCourseWare

18.950 Differential Geometry
Fall 2008

18.950 Homework 7

1. (10 points) Complete the proof of the existence of local normal coordinates (Corollary 21.3). You can use the beginning sketched in the lecture (in which case you need to summarize it for the benefit of the grader), or roll your own.

2. (4 points) Let $f : U \to \mathbb{R}^{n+1}$ be a hypersurface patch. Assume that $0 \in U$, and that the second fundamental form satisfies $g_{ij}(x) = g_{ij}(-x)$. Prove that then, the metric is in normal coordinates around that point.

3. (6 points) Suppose that $f : U \to \mathbb{R}^3$ is a surface patch, which is in normal coordinates at x_0. Write down a simple formula for the Gauss curvature at that point.