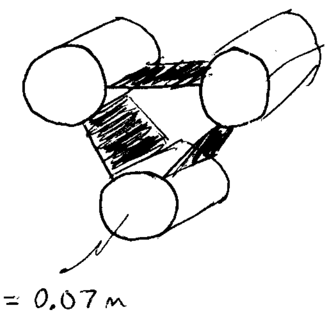
2.016 Hydrodynamics

PS #2 Fall 2005

1.



The AUV can carry M and be fully submerged but not sink.

$r_i = 0.07\,m$

$r_o = 0.08\,m$

$l = 2.0\,m$

aluminum, $s.g. = 2.7 = \dfrac{\rho_{Al}}{\rho_{H_2O}} = \dfrac{\rho_{Al}}{1000\,kg/m^3} \Rightarrow \rho_{Al} = 2700\,kg/m^3$

$$B = \rho \forall g = (1025\,\tfrac{kg}{m^3})(3\cdot\pi(0.08\,m)^2 \cdot 2.0\,m)(9.8\,m/s^2) = 1212\,N$$

$$M_{AUV} = \rho_{Al}\forall_{Al} = \rho_{Al}\,3\pi(r_o^2 - r_i^2)\,l = 2700\,kg/m^3 \cdot \pi(0.08\,m^2 - 0.07\,m^2)\cdot 2.0\,m = \underline{76.2\,kg}$$

$$B = mg + Mg$$

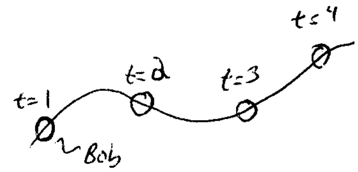
$$1212\,N = 76.2\,kg \cdot 9.8\,m/s^2 + M \cdot 9.8\,m/s^2$$

$$\boxed{M = 47.5\,kg}$$

2.016 PS #2

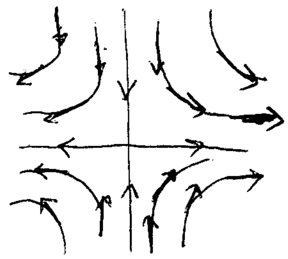
2\. a) continuum hypothesis = distance between molecular collisions is small compared to our length scale of observation.

b) A pathline connects all the points that "Bob the Fluid Blob" passes through over some period of time.



Bob's path over a period of time.

A streamline is drawn tangent to the flow for some snapshot in time



streamlines at one instant in time

If the flow is steady, then the streamlines do not change over time, and Bob will follow the path of one of the streamlines.

c)
$$(\vec{V}\cdot\vec{\nabla})\vec{V} = \left((u,v,w)\cdot\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\right)\vec{V}$$

$$= \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\vec{V}$$

$$= u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

$$= \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\hat{i} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)\hat{j} + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)\hat{k}$$

This step is true because flow is irrotational; $\vec{\nabla}\times\vec{V} = 0$ implies $\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$, $\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$, $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$

$$= \left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}\right)\hat{i} + \left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial y}\right)\hat{j} + \left(u\frac{\partial u}{\partial z} + v\frac{\partial v}{\partial z} + w\frac{\partial w}{\partial z}\right)\hat{k}$$

$$= \left(\frac{\partial}{\partial x}\left(\tfrac{1}{2}u^2\right) + \frac{\partial}{\partial x}\left(\tfrac{1}{2}v^2\right) + \frac{\partial}{\partial x}\left(\tfrac{1}{2}w^2\right)\right)\hat{i} + \left(\frac{\partial}{\partial y}\left(\tfrac{1}{2}(u^2+v^2+w^2)\right)\right)\hat{j} + \left(\frac{1}{2}\frac{\partial}{\partial z}(u^2+v^2+w^2)\right)\hat{k}$$

$$= \frac{1}{2}\vec{\nabla}(u^2+v^2+w^2)$$

$$= \frac{1}{2}\vec{\nabla}(\vec{V}\cdot\vec{V})$$

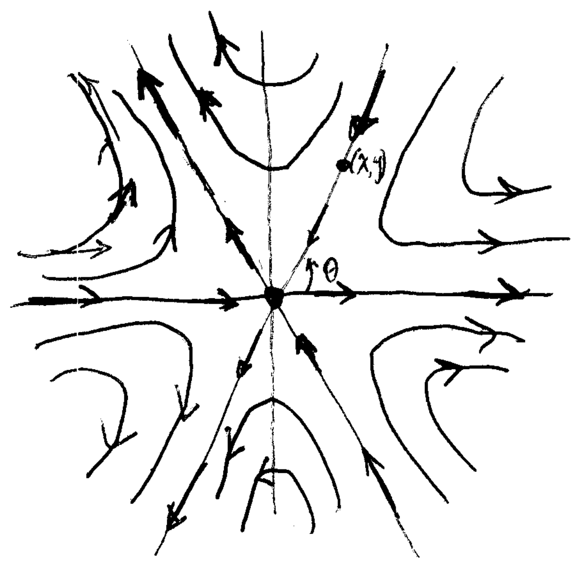
2.016 Hydrodynamics

3a) $\vec{V} = (x^2 - y^2)\,\hat{i} + (-2xy)\,\hat{j}$

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}(x^2-y^2) + \frac{\partial}{\partial y}(-2xy) = 2x - 2x = 0 \quad \checkmark \quad \text{Yes.}$$

b) Yes, the flow is steady (in the Eulerian sense) since there is no time dependence.

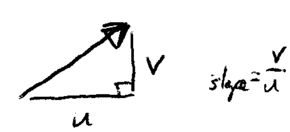
c)



d) The point (x,y) is on the line.

$$\text{Slope} = \tan\theta = \frac{y-0}{x-0} = \frac{y}{x} = \underbrace{\frac{v}{u}}_{}$$

slope of a streamline is tangent to the velocity vector

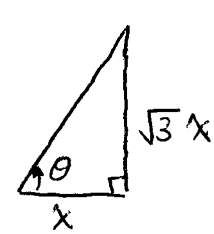


$$\frac{y}{x} = \frac{-2xy}{x^2-y^2}$$

$$x^2y - y^3 = -2x^2y$$

$$3x^2y - y^3 = 0$$

$$(3x^2 - y^2)\,y = 0$$



$$\boxed{\theta = 60^\circ}$$

y=squareroot(3)*x is the equation for the line.

$$\underbrace{(\sqrt{3}x - y)}_{}(\sqrt{3}x + y)\,y = 0$$

$y = \sqrt{3}x$ the equation for

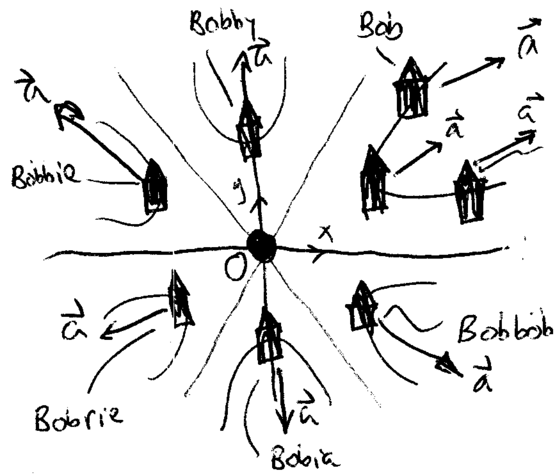
2.016 Hydrodynamics

3 f) $\vec{\nabla} \times \vec{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$

$= \left((-2y) - (-2y) \right) \hat{k}$

$= 0 \hat{k}$ ✓ Yes, the flow is irrotational.

g)



since the flow is irrotational, Bob does not spin about his center.

For Bob and all his friends, they are all pushed away from the origin. Therefore, the pressure must be highest at the origin.

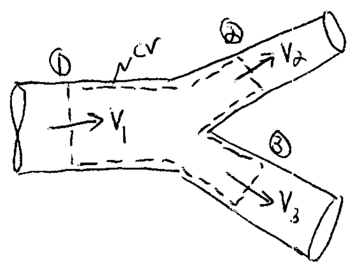
The velocity is zero at the origin, and the pressure is highest there. Does this agree with what Bernoulli's equation tells us?

$$P + \frac{1}{2}\rho V^2 + \underbrace{\rho g z} = \text{constant}$$

you can ignore this, since the flow is in the x-y plane.

2.016 PS #2

4.



$d_1 = 0.02$ m
$d_2 = 0.015$ m
$d_3 = 0.012$ m
$V_1 = 1.5$ m/s
$\dot{m}_2 = \dot{m}_3$

Find V_2, V_3.

By conservation of Mass:

$$\underbrace{\frac{d}{dt}\int_{CV} \rho \, d\forall}_{\text{rate of mass accumulating } = 0} + \underbrace{\int_{CS} \rho \vec{u} \cdot d\vec{A}}_{\text{flux out of the control volume } (\dot{m} = \rho V A)} = 0$$

$$0 - \rho V_1 \pi \frac{d_1^2}{4} + \dot{m}_2 + \underbrace{\dot{m}_3}_{=\dot{m}_2} = 0$$

$$\dot{m}_2 = \dot{m}_3 = \frac{1}{2} \rho V_1 \pi \frac{d_1^2}{4}$$

$$\dot{m}_2 = \cancel{\rho} V_2 \cancel{\pi} \frac{d_2^2}{\cancel{4}} = \frac{1}{2} \cancel{\rho} V_1 \cancel{\pi} \frac{d_1^2}{\cancel{4}}$$

$$\boxed{V_2 =} \frac{1}{2} V_1 \left(\frac{d_1}{d_2}\right)^2 = \frac{1}{2} \cdot 1.5 \text{ m/s} \cdot \left(\frac{0.020 \text{ m}}{0.015 \text{ m}}\right)^2 = \boxed{1.3 \text{ m/s}}$$

$$\boxed{V_3 =} \frac{1}{2} V_1 \left(\frac{d_1}{d_3}\right)^2 = \frac{1}{2} \cdot 1.5 \text{ m/s} \cdot \left(\frac{0.020 \text{ m}}{0.015 \text{ m}}\right)^2 = \boxed{2.1 \text{ m/s}}$$

2.016 PS #2

5. a) $P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

b) steady, incompressible, inviscid flow along a streamline, with no work done on the fluid

c) Yes. since the fluid is incompressible, the volume flow rate at any section must be the same.

d) NO. Bernoulli's equation can not be applied across the fan. The propellar does work on the fluid to increase the static pressure, P. Suppose $A_1 = A_2$ & $z_1 = z_2$, then $V_1 = V_2$ but $P_2 > P_1$ due to the fan.