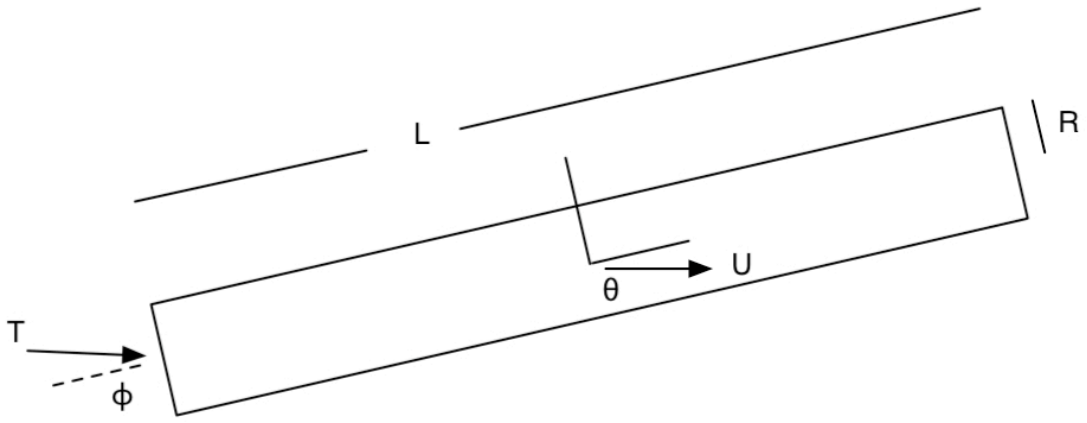
2.016 HW #5
Out: October 18, 2005
Due: October 25, 2005

1) Expand the following for i=1,2,3, j=1,2,3, and k=1,2,3, and write them in vector notation. Recall for tensor notation, letters that appear only once imply expansion into a vector, and repeated letters imply summation. For example: $u_i = \left[u_1, u_2, u_3\right] = \vec{u}$
   a. u_i
   b. $u_i v_i$
   c. $\dfrac{\partial u_i}{\partial x_i}$
   d. $u_i \dfrac{\partial v_k}{\partial x_i}$
   e. $\varepsilon_{ijk} \dfrac{\partial u_k}{\partial x_j}$

2) Consider a submarine built very poorly, such that the axis of the propeller makes an angle ϕ with the axis of the cylindrical submarine fuselage, as shown. If the propeller generates thrust, T, to drive the sub at steady-state speed, U, use inviscid flow theory (e.g. added mass) to find the trim angle, θ. (Note: despite my misleading artwork, $\phi \neq \theta$) Assume the length of the sub is much greater than the radius, $(R << L)$.



3) The equation, $\eta = a\cos(kx - \omega t)$, is for a plane progressive wave moving to the [left] [right]. Draw a picture of the wave at time t = 0 sec and time t = 1 sec, and label the distance and direction one of the peaks has traveled in that time.

4) You run several experiments in a 100 m long, 3 m wide, 2 m deep towing tank in order to investigate the shallow- and deep-water approximations to the dispersion relationship. In each experiment, plane progressive waves are sent down the length of the tank using a wave paddle. For each wavelength given, calculate the frequency ω using the dispersion relationship and again using the appropriate approximation. What is the error of the approximation $\left(error = \frac{exact - approx}{exact} \right)$?

   a. $\lambda = 125$ m
   b. $\lambda = 21$ m
   c. $\lambda = 12.6$ m
   d. $\lambda = 6.28$ m
   e. $\lambda = 2.1$ m

   I suggest making a table in Excel for this exercise, because you will use some of the stuff in the table to do the next problem, and because the hyperbolic trig functions are a pain to do by hand. Write down the equations you use at the top of each column, so I know how you calculated each value.

5) Now compare the exact phase speed with the phase speed you'd calculate using the approximate frequencies. What is the percent error?

6) Finally, compare the exact group speed with the group speed you would approximate using the group speed approximations and the approximate phase speeds you found using the approximate frequencies. (Geez, that a lot of approximates!) What is the percent error?

7) Sketch the orbits of particles at various heights for the 21 m wavelength wave.