

2.016 Hydrodynamics

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0.1 Added Mass Forces and Moments

The added mass forces and moments are given by the following, for $i = 1, 2, 3, 4, 5, 6$ and $j, k, l = 1, 2, 3$:

$$F_j = -\dot{U}_i m_{ji} - \epsilon_{jkl} U_i \Omega_k m_{li}$$

$$M_j = -\dot{U}_i m_{j+3,i} - \epsilon_{jkl} U_i \Omega_k m_{l+3,i} - \epsilon_{jkl} U_i U_k m_{li}$$

$$F_1 = -\dot{U}_1 m_{11} - \dot{U}_2 m_{12} - \dot{U}_3 m_{13} - \dot{U}_4 m_{14} - \dot{U}_5 m_{15} - \dot{U}_6 m_{16}$$

$$- U_1 \Omega_2 m_{31} - U_2 \Omega_2 m_{32} - U_3 \Omega_2 m_{33} - U_4 \Omega_2 m_{34} - U_5 \Omega_2 m_{35} - U_6 \Omega_2 m_{36}$$

$$+ U_1 \Omega_3 m_{21} + U_2 \Omega_3 m_{22} + U_3 \Omega_3 m_{23} + U_4 \Omega_3 m_{24} + U_5 \Omega_3 m_{25} + U_6 \Omega_3 m_{26}$$

$$F_2 = -\dot{U}_1 m_{21} - \dot{U}_2 m_{22} - \dot{U}_3 m_{23} - \dot{U}_4 m_{24} - \dot{U}_5 m_{25} - \dot{U}_6 m_{26}$$

$$- U_1 \Omega_3 m_{11} - U_2 \Omega_3 m_{12} - U_3 \Omega_3 m_{13} - U_4 \Omega_3 m_{14} - U_5 \Omega_3 m_{15} - U_6 \Omega_3 m_{16}$$

$$+ U_1 \Omega_1 m_{31} + U_2 \Omega_1 m_{32} + U_3 \Omega_1 m_{33} + U_4 \Omega_1 m_{34} + U_5 \Omega_1 m_{35} + U_6 \Omega_1 m_{36}$$

$$F_3 = -\dot{U}_1 m_{31} - \dot{U}_2 m_{32} - \dot{U}_3 m_{33} - \dot{U}_4 m_{34} - \dot{U}_5 m_{35} - \dot{U}_6 m_{36}$$

$$- U_1 \Omega_1 m_{21} - U_2 \Omega_1 m_{22} - U_3 \Omega_1 m_{23} - U_4 \Omega_1 m_{24} - U_5 \Omega_1 m_{25} - U_6 \Omega_1 m_{26}$$

$$+ U_1 \Omega_2 m_{11} + U_2 \Omega_2 m_{12} + U_3 \Omega_2 m_{13} + U_4 \Omega_2 m_{14} + U_5 \Omega_2 m_{15} + U_6 \Omega_2 m_{16}$$

$$M_1 = -\dot{U}_1 m_{41} - \dot{U}_2 m_{42} - \dot{U}_3 m_{43} - \dot{U}_4 m_{44} - \dot{U}_5 m_{45} - \dot{U}_6 m_{46}$$

$$- U_1 \Omega_2 m_{61} - U_2 \Omega_2 m_{62} - U_3 \Omega_2 m_{63} - U_4 \Omega_2 m_{64} - U_5 \Omega_2 m_{65} - U_6 \Omega_2 m_{66}$$

$$+ U_1 \Omega_3 m_{51} + U_2 \Omega_3 m_{52} + U_3 \Omega_3 m_{53} + U_4 \Omega_3 m_{54} + U_5 \Omega_3 m_{55} + U_6 \Omega_3 m_{56}$$

$$- U_1 U_2 m_{31} - U_2 U_2 m_{32} - U_3 U_2 m_{33} - U_4 U_2 m_{34} - U_5 U_2 m_{35} - U_6 U_2 m_{36}$$

$$+ U_1 U_3 m_{21} + U_2 U_3 m_{22} + U_3 U_3 m_{23} + U_4 U_3 m_{24} + U_5 U_3 m_{25} + U_6 U_3 m_{26}$$

$$M_2 = -\dot{U}_1 m_{51} - \dot{U}_2 m_{52} - \dot{U}_3 m_{53} - \dot{U}_4 m_{54} - \dot{U}_5 m_{55} - \dot{U}_6 m_{56}$$

$$- U_1 \Omega_3 m_{41} - U_2 \Omega_3 m_{42} - U_3 \Omega_3 m_{43} - U_4 \Omega_3 m_{44} - U_5 \Omega_3 m_{45} - U_6 \Omega_3 m_{46}$$

$$+ U_1 \Omega_1 m_{61} + U_2 \Omega_1 m_{62} + U_3 \Omega_1 m_{63} + U_4 \Omega_1 m_{64} + U_5 \Omega_1 m_{65} + U_6 \Omega_1 m_{66}$$

$$- U_1 U_3 m_{11} - U_2 U_3 m_{12} - U_3 U_3 m_{13} - U_4 U_3 m_{14} - U_5 U_3 m_{15} - U_6 U_3 m_{16}$$

$$+ U_1 U_1 m_{31} + U_2 U_1 m_{32} + U_3 U_1 m_{33} + U_4 U_1 m_{34} + U_5 U_1 m_{35} + U_6 U_1 m_{36}$$

$$M_3 = -\dot{U}_1 m_{61} - \dot{U}_2 m_{62} - \dot{U}_3 m_{63} - \dot{U}_4 m_{64} - \dot{U}_5 m_{65} - \dot{U}_6 m_{66}$$

$$- U_1 \Omega_1 m_{51} - U_2 \Omega_1 m_{52} - U_3 \Omega_1 m_{53} - U_4 \Omega_1 m_{54} - U_5 \Omega_1 m_{55} - U_6 \Omega_1 m_{56}$$

$$+ U_1 \Omega_2 m_{41} + U_2 \Omega_2 m_{42} + U_3 \Omega_2 m_{43} + U_4 \Omega_2 m_{44} + U_5 \Omega_2 m_{45} + U_6 \Omega_2 m_{46}$$

$$- U_1 U_1 m_{21} - U_2 U_1 m_{22} - U_3 U_1 m_{23} - U_4 U_1 m_{24} - U_5 U_1 m_{25} - U_6 U_1 m_{26}$$

$$+ U_1 U_2 m_{11} + U_2 U_2 m_{12} + U_3 U_2 m_{13} + U_4 U_2 m_{14} + U_5 U_2 m_{15} + U_6 U_2 m_{16}$$