2.032 DYNAMICS Fall 2004

Problem Set No. 5

Out: Wednesday, October 13, 2004

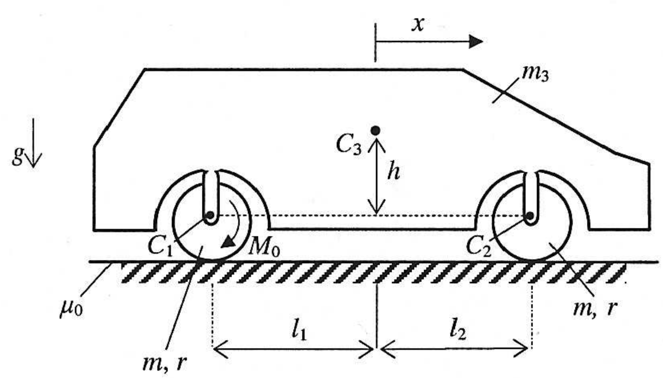
Due: Wednesday, October 20, 2004 *at the beginning of class*

Problem 1

Consider the motion of a car, as shown in the figure below. (Out of the four wheels only two are visible.) The mass of the body of the car is m_3, and each wheel has radius r and mass m. The coefficient of static friction is μ_0 i.e., the tangential force between the wheel and the ground is never greater than μ_0 times the normal force. The torque transmitted from the body of the car to the rear wheel set is M_0.

(a) Assuming pure rolling, use the work-energy principle to find the acceleration of the car.

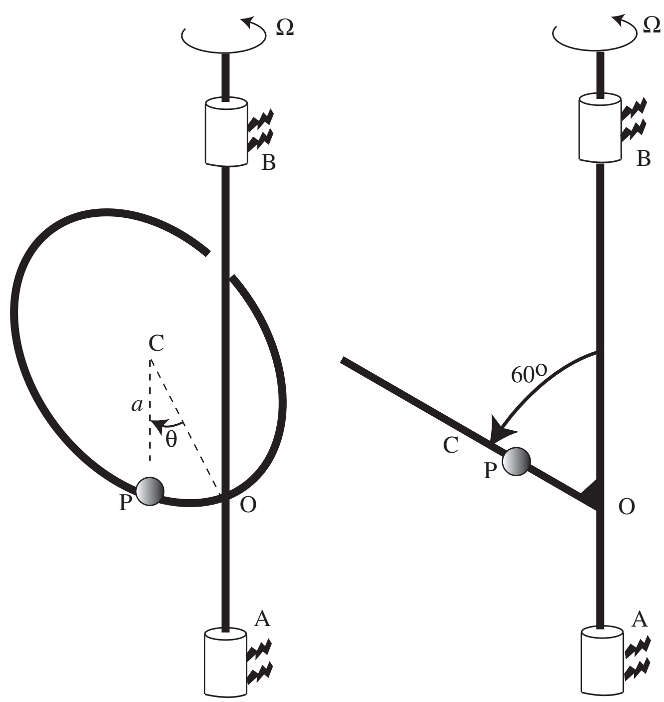
(b) What is the largest M_0 for which the car does not slip?



Problem 2 (adapted from PhD Qualifying Exam 2003)

A circular ring of radius a is fixed to a vertical shaft AB at O. The plane of the ring is inclined at $60°$ to the vertical as shown below so that the included angle COB is $60°$, where C is the center of the ring. A particle P of mass m is free to slide without friction on the ring. The position of the particle is measured by the included angle OCP, denoted θ as shown. The shaft is driven at *constant* angular velocity Ω.

Find in terms of θ and $\dot{\theta}$ the velocity of P.



Problem 3 (adapted from Crandall et al., 4-20)

A rigid cone with apex half angle α rolls steadily without slip on a horizontal surface so that it precesses about the Z axis at a constant angular rate Ω. The cone has mass M and principal moments of inertia I_1, I_1, and I_3 at the tip.

(a) Determine the angular momentum of the cone.

(b) What forces and torques are required to maintain this motion?

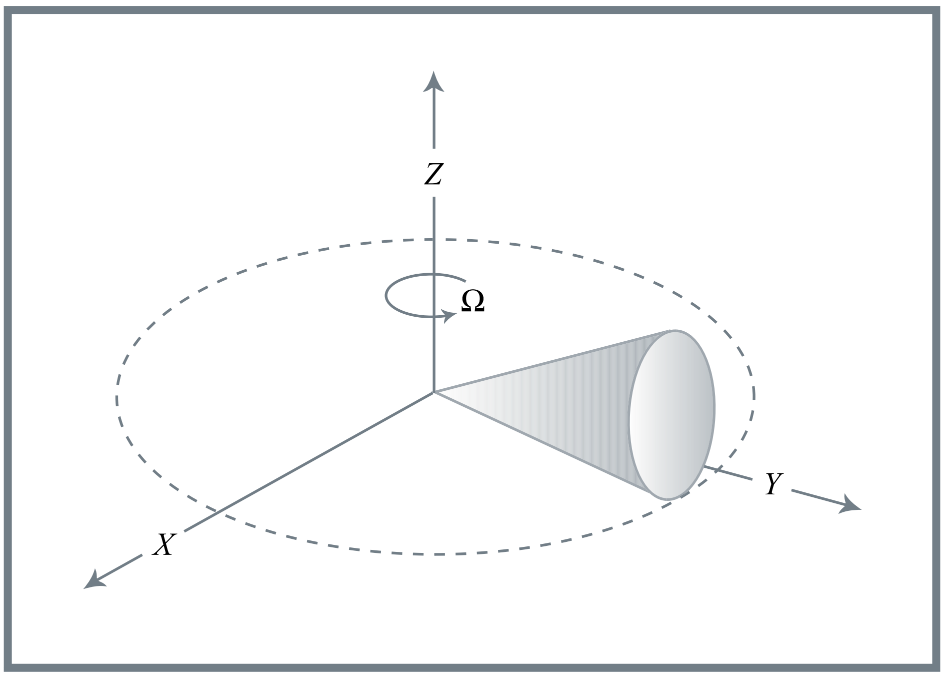


Figure by OCW. After problem 4-20 in Crandall, S. H., et al. *Dynamics of Mechanical and Electromechanical Systems*. Malabar, FL: Krieger, 1982.

Problem 4

The heavy disk of mass M and centroidal principal moments of inertia $I_1 = I_2, I_3$ rolls without slipping in contact with the inclined plane, as shown below. A ball joint at O holds the end of the massless shaft in place. The shaft makes angle β with the inclined plane. Derive an expression for the kinetic energy of the disk.