

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering
2.04A Systems and Controls
Spring 2013

Problem Set #2

Posted: Thursday, Feb. 14, '13

Due: Thursday, Feb. 21, '13

1. Obtain the inverse Laplace transform of the following frequency-domain expressions:

1.a) $F_1(s) = -\frac{(4s - 10)}{s(s + 2)(s + 5)}$;

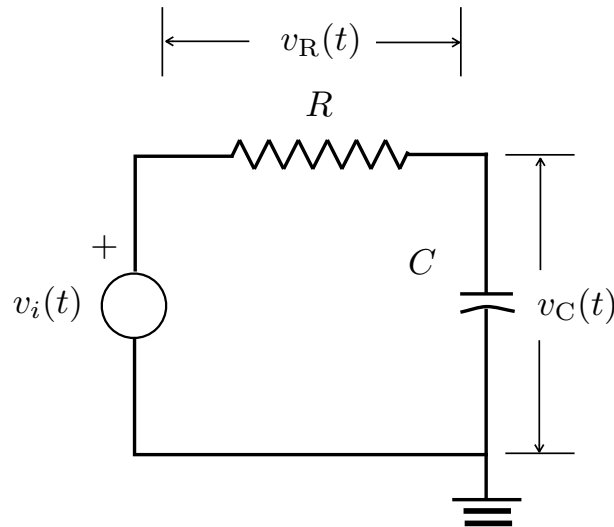
1.b) $F_2(s) = \frac{4}{s^2(s^2 + 4)}$;

1.c) $F_3(s) = \frac{s^3 - 3s^2 + s + 2}{s}H(s)$, where $H(s)$ is a well-behaved function of s ,

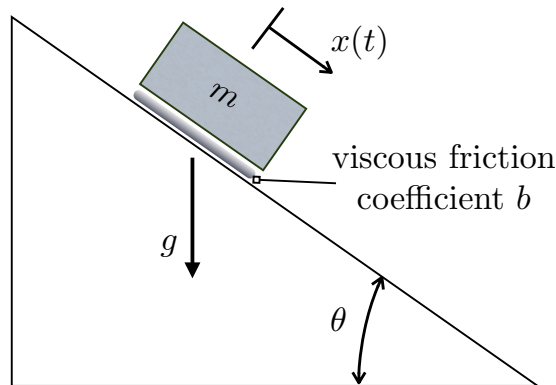
whose inverse Laplace transform is $h(t)$.

2. Obtain the transfer function of problem 2.a in the Problem Set #1 by Laplace transforming the equations of motion that you derived previously. (If you don't have copy of your solution, you can find the equations on motion in the solutions to Problem Set #1) The input to the system is the force $f(t)$ and the output the rotation angle $\theta(t)$.
3. Consider the resistor-capacitor (RC) circuit drawn below, where the input is a voltage source $v_i(t)$ and the output is the voltage $v_C(t)$ across the capacitor. Our purpose in this problem is to establish the *analogy* between this system and the flywheel system (with torque as input and angular velocity as output.)
- 3.a) Using the voltage-divider principle, derive the RC system's transfer function $V_C(s)/V_i(s)$.
- 3.b) Using the transfer function, derive the impulse response and the step response of the RC system in the time domain.
- 3.c) Compare with the respective flywheel responses, and identify "equivalent" quantities.
- 3.d) Now we change the system so that its output is the voltage $v_R(t)$ across the resistor. What is the new transfer function $V_R(s)/V_i(s)$?

- 3.e) What is the relationship between the impulse and step responses of the RC system with $v_R(t)$ as output to the respective responses with $v_C(t)$ as the output?



4. A mass m is initially at rest on an infinitely long ramp surface and is released abruptly at time $t = 0$. We assume that the motion of the mass is subject to viscous friction from a lubricant applied at the interface with the ramp, and that the viscous friction coefficient is b . The ramp incline angle is θ and the distance along the ramp that the mass will slide as function of time $t > 0$ is $x(t)$.



- 4.a) We model this physical situation as a system with the weight of the mass as input ("turned on" at $t = 0$ when the mass is released, constant thereafter) and the velocity $v(t) = \dot{x}(t)$ as output. What is the transfer function of this system?
- 4.b) Use the transfer function to derive an expression for the velocity as function of time t , for $t > 0$. Can you identify one (or more) time constant(s)?

- 4.c) As time $t \rightarrow \infty$, show that the velocity approaches a constant value. This is known as the system's "steady state" in general, and in the specific case of an object falling due to its own weight while it is subject to friction, as "terminal velocity." Derive an expression for the terminal velocity in this system.
- 4.d) Identify analogies between this system and the RC system of the previous problem, as well as the flywheel system from the laboratory.
- 4.e) Now consider the same physical situation, except we define the output as the mass travel $x(t)$. What is the transfer function of this new system?
- 4.f) Use at least *two* methods to derive an expression for $x(t)$, and verify that the two methods yield consistent answers.

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