1.138J/2.062J/18.376J, WAVE PROPAGATION
Fall, 2006 MIT
FINAL EXAMINATION, DEC 4-DEC 11(12 am)
PLEASE CHOOSE ANY FOUR QUESTIONS. IF YOU TRY ALL FIVE, PLEASE IDENIFY FOUR FOR GRADING.

Please abide by the following rules of this exam, strictly.

- You can use handouts, **your own** notes, homework and mathematical handbooks. Do not use any other references, printed or handwritten.
- If you have any question regarding the questions and the exam, ask me only . Do not ask others even just for clarification of the exam. In case the questions are of general interest, I will inform all of you by E-mail.

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1 Sound scattered by a wavy surface.

Sound scattered by the sea waves is of interest in oceanography. As a simpler problem let a train of monochromatic plane waves be incident obliquely on a wavy wall widhci is perfectly rigid and reflective. All sound-wave crests are parallel the wave crests of the wall so that motion is two-dimensional in x(horizonatal) and y(vertical). The total potential is governed by with

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{C^2}\frac{\partial^2 \phi}{\partial t^2} \tag{H.1.1}$$

where C =constant. The incident wave is given by

$$\phi_I = \frac{A}{2} e^{i(\alpha x - \beta y) - i\omega t} + c.c. \tag{H.1.2}$$

where $\omega = kC$ is the speed of sound in air. The slightly wavy is given by

$$y = a \sin Kx \tag{H.1.3}$$

where $Ka \ll 1$. Find the boundary condition first and then the approximate solution by straightforward perturbations to show the leading-order effect of the waviness. Discuss the solution and identify the condition for possible resonance where the scattered wave is no longer a small perturbation.

How would you proceed to construct a multiple-scale theory valid at and near resonance? (Do not go into details unless you want to make it a research topic.)

2 Can energy be extracted from sea waves ?

Consider the following idealized wave-power extractor. A heavy gate stands vertically across the entire width of a long channel of rectangular cross section and water depth h. The gate has the mass M per unit width of the channel and can slide back and forth horizontally without friction, and is supported at the back by a spring of elastic costant K and a power extractor. The extractor exerts a damping force on the gate proportional to the velocty of the gate

$$-c\frac{dX}{dt} \tag{H.2.4}$$

where $X(t)$ is the displacement of the gate and c the extraction rate. Both K and c refer to unit width of the channel.

A plane wave of surface displacement

$$\zeta = Ae^{-i(kx+\omega t)} \tag{H.2.5}$$

arrives from $x \sim \infty$. Find the best rate c which maximizes the power extracted from the incident wave. Compare the extracted power with the power flux per unit width of the wave $\frac{1}{2}\rho g A^2 C_g$ where $C_g = \sqrt{gh}$ for long waves.

To simplify the calculations, assume long waves in shallow water and small amplitude motion for both waves and the gate so that the linearized long-wave theory suffices.

3 Ship waves in shallow-water

In a stationary frame of reference, the linearizied equations governing long waves in shallow water of constant depth h are:

$$\frac{\partial \zeta}{\partial t} + h\nabla \cdot \vec{u} = 0 \tag{H.3.6}$$

and

$$\frac{\partial \vec{u}}{\partial t} = -g\nabla\zeta \tag{H.3.7}$$

A slender ship moves at the constant speed U in the negative x direction. Transform to the moving coordinate system so that the ship is stationary and there is a current of velocity U along the positve x direction. Show that in the ship-bound coordinates, the velocity disturbance due to the ship is governed by

$$\left(1 - \frac{U^2}{gh}\right)\phi_{xx} + \phi_{yy} = 0 \tag{H.3.8}$$

for the velocity disturbance potential $\vec{u} = \nabla\phi$.

Let the ship be modelled by a vertical strut (bridge-pier) whose hull is

$$y = \pm Y(x) \tag{H.3.9}$$

show that the exact no-flux condition is,

$$\frac{\partial \phi}{\partial y} = \pm \left(U + \frac{\partial \phi}{\partial x} \right) \frac{\partial Y}{\partial x}, \quad \text{on} \quad y = \pm Y(x). \tag{H.3.10}$$

Approximate it to include the leading order effect of the ship hull.

Solve for the potential for a supercritical flow where $U > \sqrt{gh}$ (Mach Number > 1). Describe the wave picture.

Find the hydrodynamic pressure on the hull and then get the wave resistance suffered by the ship by integrating the pressure.

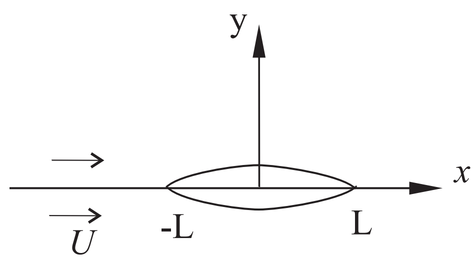

Figure 1: Steady flow past a slender ship

4 Tsunami due to land slide

Tsunamis can be generated by land slides. Consider a two dimensional lake $x > 0, -h < z < 0$ bounded on the left by a mountain range. At $t = 0$ a slice of the mountain falls into the lake. Let us model the landslide by the initial conditions

$$\zeta(x,0) = \frac{2S}{D}\left(1 - \frac{x}{D}\right), \quad 0 < x < D; \quad \zeta(x,0) = 0, \quad D < x < \infty \tag{H.4.11}$$

and

$$\phi(x,0) = 0 \quad x > 0. \tag{H.4.12}$$

S is the equivalent volume of land mass falling into water. Find the leading waves far away from the mountain, the time history, the maximum amplitude etc.

If on the far side of the mountain the lake is bounded by a vertical dam at $x = L \ggg D$. Find the expression of the dynamic pressure on the dam. Discuss the result. Hint: Apply the method of images.

5 Sound in a randomly rough waveguide

A plane wave of sound

$$\phi_I = Ae^{i(kx-\omega t)} \tag{H.5.13}$$

arrives from the end $x \sim -\infty$ of a long channel of mean width B. In $x < 0$ the walls are smooth so that the walls are described by

$$y = 0, B \tag{H.5.14}$$

where B=constant. In $0 < x < \infty$ one wall is rough so that it is described by

$$y = B + \epsilon b(x), \quad \epsilon \ll 1. \tag{H.5.15}$$

For simplicity let us assume that $b(x)$ is a homogeneous random function of x with zero mean and prescribed correlation function $C(|x - x'|)$

$$\langle b(x) \rangle = 0, \tag{H.5.16}$$

$$\langle b(x) b(x') \rangle = D^2 C(|x - x'|) \tag{H.5.17}$$

where $C(0) = 1$ and ϵD is the maximum root-mean-square height.

Find the rule governing the slow attenuation of A over long distances $\epsilon^2 x = O(1)$.