1.138J/2.062J/18.376J Wave Propagation

Take-Home Exam

This is a *closed-book* exam. You may use *only* your own class notes, problem sets and the lecture notes posted on the 1.138J/2.062J/18.376J website. *You are not allowed to discuss this exam with anyone else*.

Problem 1 (10 points)

A long, uniform taut string (mass per unit length ρ, tension T) along $-\infty < x < \infty$ is supported on an elastic foundation of stiffness α, and a point mass M is attached at $x = 0$.

Suppose that a time-harmonic vertical force

$$F \cos \Omega t$$

is applied to the mass at $x = 0$.

Determine the steady-state displacement response of the string for $-\infty < x < \infty$.

Problem 2 (10 points)

The propagation of free uni-directional surface waves of small amplitude on moderately shallow water is governed by the equation

$$\frac{\partial \eta}{\partial t} + c_0 \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0,$$

where $\eta(x,t)$ is the free-surface elevation and c_0 and β are constants.

(a) Suppose that an external localized pressure disturbance traveling with constant speed V acts on the free surface. Determine the wavenumber(s) of the excited steady-state radiating wave(s), depending on the forcing speed V. Sketch the position of these waves relative to the forcing. (Take $c_0 > 0$ and consider $\beta > 0$ and $\beta < 0$ as well as $V > 0$ and $V < 0$.)

(b) Suppose at $t = 0$ a localized initial wave disturbance is introduced in the vicinity of $x = 0$. Sketch qualitatively the time history of the response for $t > 0$ at a fixed station $x = L > 0$, far from the region of the initial disturbance. Sketch qualitatively a snapshot of the disturbance for $-\infty < x < \infty$ at time $t = T$, long after the initial excitation. Justify your answers. (Again, $c_0 > 0$ and consider $\beta > 0$ and $\beta < 0$.)

Problem 3 (10 points)

Consider a long uniform string of mass per unit length ρ, split into two pieces. The two halves are attached to a massless ring which slides vertically without friction on a fixed rod at $x = 0$. The left string half ($x \leq 0$) is taut with tension T while the right string half ($x \geq 0$) is taut with tension T'.

Suppose that a traveling wave of frequency ω comes in from the negative x direction. Compute the reflection and transmission coefficients.

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