Buckling of Stiffened Panels 1
overall buckling vs plate buckling
PCCB Panel Collapse Combined Buckling

Various estimates have been developed to determine the minimum size stiffener to insure the plate buckles while the stiffener remains straight. this is equivalent to insuring that plate buckling occurs before overall buckling.

Timoshenko does so by calculating k in $\sigma_{cr} := k \cdot \frac{\pi^2 \cdot D}{b^2 \cdot t}$ and observing the value of γ which results in a critical stress above that which will cause plate buckling alone. where;

$$\gamma := \frac{\text{flexural\_rigidity\_of\_combined\_section}}{\text{flexural\_rigidity\_of\_plate}} \qquad \gamma := \frac{E \cdot I_x}{D \cdot b}$$ where;

I_x is the inertia of the plate with the attached plate associated with individual stiffener.

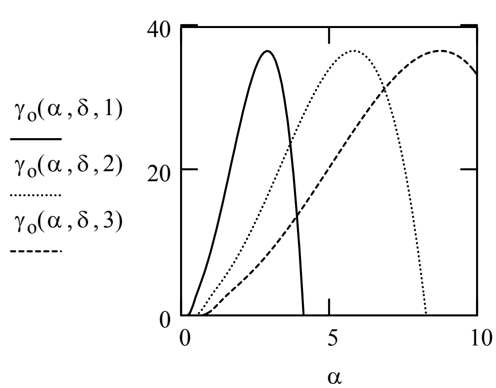
Bleich pg 365, 367, for plates with longitudinal stiffness determines minimum γ to insure the plate buckles before the stiffener (overall buckling). unfortunately Bleich uses different ratios than Hughes. Bleich uses B where Hughes uses b.

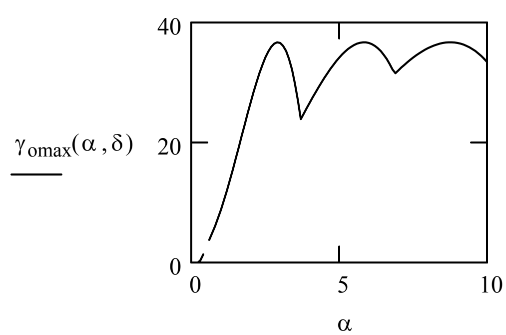
for the following: $\alpha := \frac{a}{B}$ $\qquad \delta := \frac{A_x}{B \cdot t}$ $\qquad \gamma := \frac{E \cdot I_x}{D \cdot B}$

$$\alpha := 0.1, 0.2 .. 10 \qquad \delta := 0.1 \qquad \kappa_1(\alpha, n) := n \cdot \frac{\pi}{\alpha} \cdot \sqrt{\frac{4 \cdot \alpha}{n} + 1} \qquad \kappa_2(\alpha, n) := n \cdot \frac{\pi}{\alpha} \cdot \sqrt{\frac{4 \cdot \alpha}{n} - 1}$$

$$\gamma_o(\alpha, \delta, n) := \frac{\frac{16}{\pi^2} \cdot \left(\frac{\alpha}{n}\right)^3}{\frac{1}{\kappa_1(\alpha, n)} \cdot \tanh\left(\frac{\kappa_1(\alpha, n)}{2}\right) - \frac{1}{\kappa_2(\alpha, n)} \cdot \tan\left(\frac{\kappa_2(\alpha, n)}{2}\right)} + 16 \cdot \left(\frac{\alpha}{n}\right)^2 \cdot \delta$$

$$\gamma_{omax}(\alpha, \delta) := \max\left(\begin{pmatrix} \gamma_o(\alpha, \delta, 1) \\ \gamma_o(\alpha, \delta, 2) \\ \gamma_o(\alpha, \delta, 3) \end{pmatrix}\right)$$





curve fit: one stiffener, combining Hughes and Bleich terms, N=1 stiffener

$$\gamma_{bx1}(\alpha,\delta) := \frac{22.8}{2}\cdot\alpha + \left(\frac{2.5}{2} + 16\cdot\delta\right)\cdot\alpha^2 - \frac{10.8}{2}\cdot\sqrt{\alpha}$$

$$\gamma_{bx2}(\alpha,\delta) := \frac{48.8}{2} + 112\cdot\delta\cdot\left(1 + \frac{0.5}{2}\cdot 2\cdot\delta\right) \qquad \gamma_{bx}(\alpha,\delta) := \min\left(\begin{pmatrix}\gamma_{bx1}(\alpha,\delta)\\ \gamma_{bx2}(\alpha,\delta)\end{pmatrix}\right)$$

Hughes: 13.1.4 slightly modified in terms

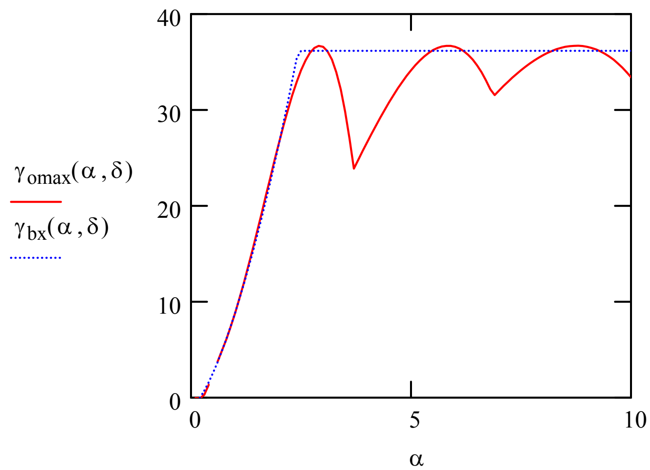


Fig. 180, Bleich

to move to Bleich relationships in Hughes terms (ratios) to match text:

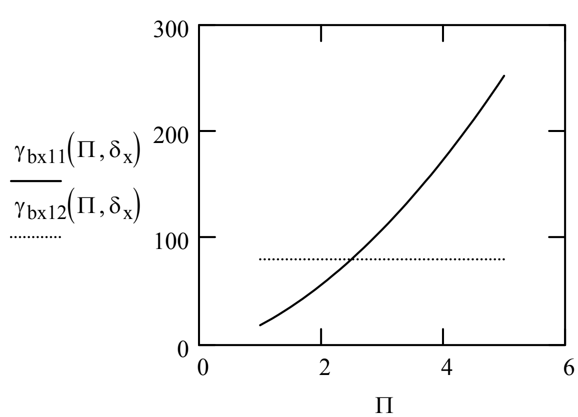
where: $\Pi := \frac{a}{B}$ $\qquad \delta_x := \frac{A_x}{b\cdot t}$ $\qquad \gamma := \frac{E\cdot I_x}{D\cdot b}$

one stiffener

$$\gamma_{bx11}(\Pi,\delta_x) := 22.8\cdot\Pi + (2.5 + 16\cdot\delta_x)\cdot\Pi^2 - 10.8\cdot\sqrt{\Pi}$$

$$\gamma_{bx12}(\Pi,\delta_x) := 48.8 + 112\cdot\delta_x\cdot(1 + 0.5\cdot\delta_x)$$

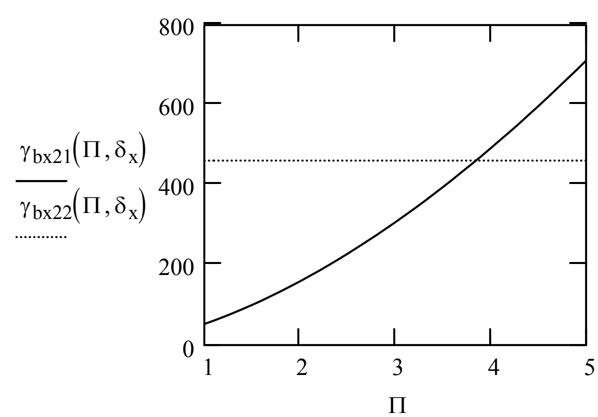
$\Pi := 1, 1.1 .. 5$



two stiffeners

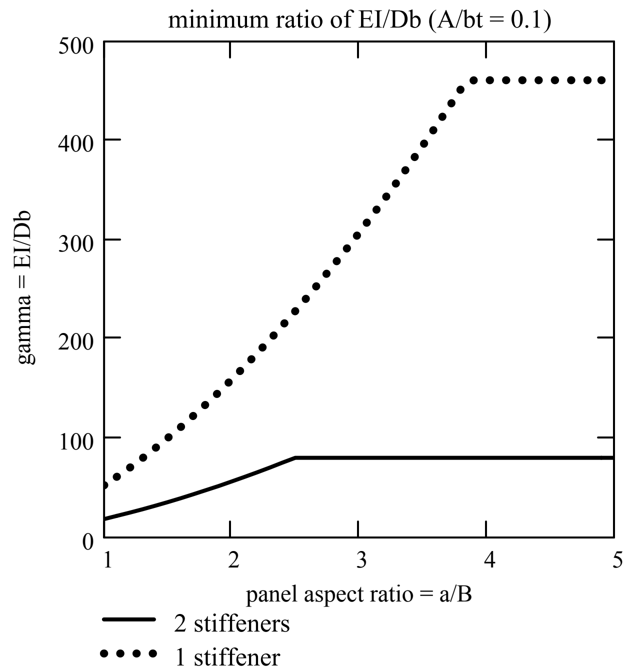
$$\gamma_{bx21}(\Pi,\delta_x) := 43.5\cdot\sqrt{\Pi^3} + 36\cdot\Pi^2\cdot\delta_x$$

$$\gamma_{bx22}(\Pi,\delta_x) := 288 + 610\cdot\delta_x + 325\cdot\delta_x^2$$



combination

$$\gamma_b(\Pi,\delta_x,N) := \mathrm{if}\left[N = 1, \min\left(\begin{pmatrix}\gamma_{bx11}(\Pi,\delta_x)\\ \gamma_{bx12}(\Pi,\delta_x)\end{pmatrix}\right), \min\left(\begin{pmatrix}\gamma_{bx21}(\Pi,\delta_x)\\ \gamma_{bx22}(\Pi,\delta_x)\end{pmatrix}\right)\right]$$



A more direct approach is to calculate the overall buckling stress and insure it is larger than the plate critical stress.

The overall buckling stress is the value at which the stiffeners reach critical stress, modeling each stiffener as a column of stiffener with attached (portion) of plate with some equivalent slenderness ratio.

$\lambda := L\_over\_\rho_{eq}$

We will continue to model the plate failure as a gradual failure i.e the center of the plate "fails" in buckling while the outer section remains effective at an *effective breadth* b_e

paradoxically, the column is "stiffer" when the plate flange (b_e) is reduced for ratios typical

of ship structure: let's first evaluate the plate and column critical stresses for a short panel. As we assumed in plate buckling (and bending) the width is such that we can model a slice independently:

the column is a stiffener with an attached plate of width b
the plate is a width b some typical scantlings:

stiffener web	stiffener flange	stiffener area	stiffener depth	panel length	breadth between stiffeners b	set above
$A_w := 0.8$	$A_f := 0.85$	$A_s := A_w + A_f$	$d := 5$	$L := 96$	$b := 30$	$t = 0.5$

calculate in terms of b_e, initially b_e = b
moment of inertia using 8.3.6 to calculate radius of gyration:

$$A_e(b_e) := A_s + b_e \cdot t \qquad C_1(b_e) := \frac{A_w \cdot \left(\frac{A_e(b_e)}{3} - \frac{A_w}{4}\right) + A_f \cdot b_e \cdot t}{\left(A_e(b_e)\right)^2} \qquad I_e(b_e) := A_e(b_e) \cdot (d)^2 \cdot C_1(b_e)$$

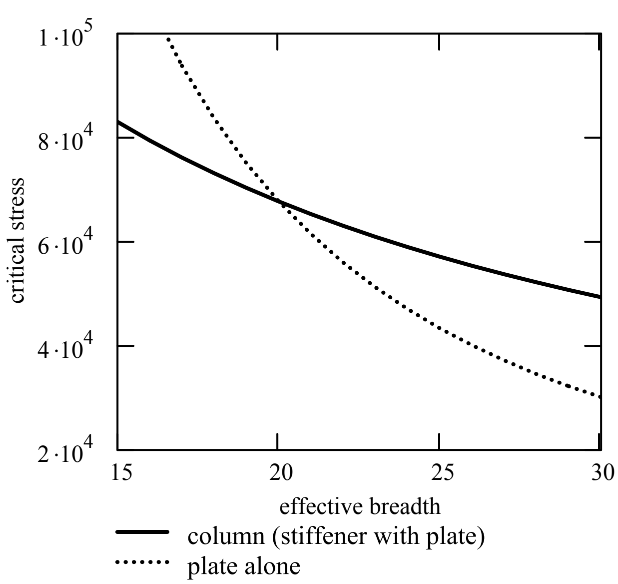
$$\rho_e(b_e) := \sqrt{\frac{I_e(b_e)}{A_e(b_e)}}$$

column critical stress: $\sigma_{e\_cr}(b_e) := \dfrac{\pi^2 \cdot E}{\left(\dfrac{L}{\rho_e(b_e)}\right)^2}$; initial value $\sigma_{e\_cr}(b) = 49341$

plate critical stress: $\sigma_{a\_cr}(b_e) := 3.615 \cdot E \cdot \left(\dfrac{t}{b_e}\right)^2$ initial value $\sigma_{a\_cr}(b) = 30125$

things are ok as column > plate => plate "fails" first. now consider increasing stress beyond $\sigma_{a\_cr}(b)$ and plate gradually fails reducing effective breadth. Note that we are using an assumption due to von Karman, that the "failed" center region has no compressive stress while the outer regions are fully effective at σ_e defined from force equilibrium as $\sigma_e(b_e) := \dfrac{\sigma_a \cdot b}{b_e}$ now consider what happens to the values of critical stress as effective breadth is reduced. the definitions above are still active; repeated here for info:

$b_e := 15 .. b$



these values are not "short" but with appropriate scantlings the result is the same. this set of values assumes yield stress is > 3x10^4 or so.

column: $\sigma_{e\_cr}(b_e) := \dfrac{\pi^2 \cdot E}{\left(\dfrac{L}{\rho_e(b_e)}\right)^2}$

plate: $\sigma_{a\_cr}(b_e) := 3.615 \cdot E \cdot \left(\dfrac{t}{b_e}\right)^2$

even though the numbers above represent a long column (plate), the analysis is only appropriate for short panels. to analyze long panels a correction is needed to λ = L_over_ρ to account for the tendency of the plate to deflect with more than one half sine wave making the column slightly stiffer (smaller slenderness ratio λ). The correction is as follows:

$$C_\pi := \frac{B}{a}\cdot\sqrt{\frac{\gamma_x(b_e)}{2\cdot\left(1+\sqrt{1+\gamma_x(b_e)}\right)}} \quad \text{or} \quad C_\pi := 1 \quad \text{whichever is less.} \quad \gamma_x(b_e) := \frac{E\cdot I_x(b_e)}{D\cdot b_e}$$

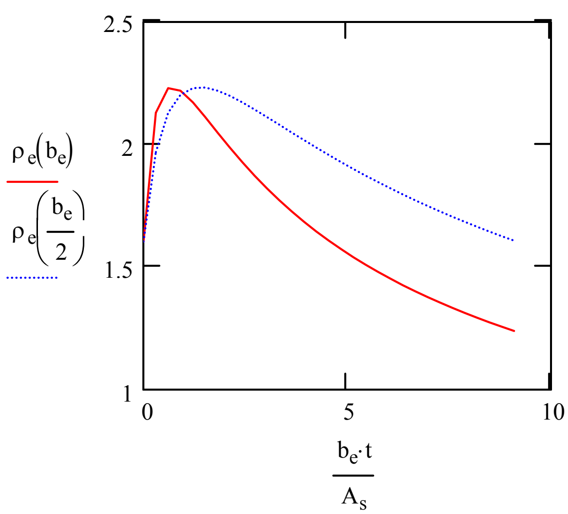
and $L\_over\_\rho_{eq} = C_\pi\cdot\frac{L}{\rho(b_e)}$ then $\sigma_{e\_cr}(b_e) := \dfrac{\pi^2\cdot E}{\left(C_\pi\cdot\dfrac{L}{\rho_e(b_e)}\right)^2}$

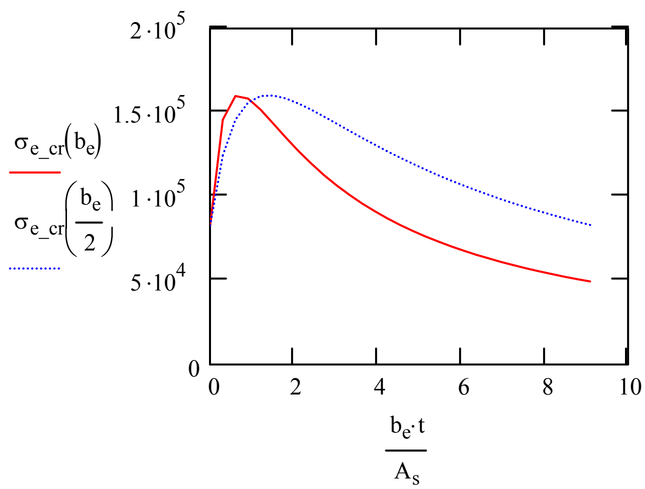
this adds a non linearity to the problem so an iterative method is used to make the initial comparison of whether the column critical stress is greater than the plate critical stress or to solve for the common critical stress. The text proposes an iterative method similar to that below for PCCB to determine the common value of critical stress. After doing so it is necessary to bring this stress (based on effective breadth) back to applied (average) stress; again using statics

$\sigma_a\cdot(b\cdot t + A_x) = \sigma_e\cdot(b_e\cdot t + A_x)$ and $\sigma_a(b_e) := \dfrac{(b_e\cdot t + A_x)}{(b\cdot t + A_x)}\cdot\sigma_{e\_cr}(b_e)$ at the effective breadth that the iteration converges.

Some calculations to validate the column becoming "stiffer" with less effective plate. Think of the location of the neutral axis or the radius of gyration as the plate is reduced. With a wide plate flange the neutral axis is close to the plate. The radius of gyration which plays in the critical stress varies as follows: (recall larger ρ => larger critical stress:

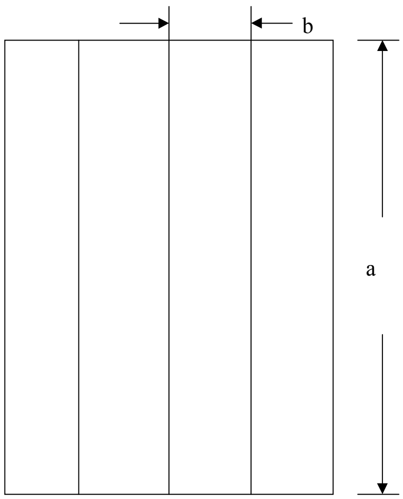
$b_e := 0 .. b$

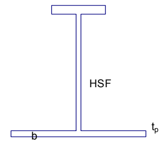




now for our design rules for PCCB Panel Collapse Combined Buckling:

- PCCB - Panel Collapse Combined Buckling
function of b_e, graphical approach





$input := 1$

input data
ignore numerical
values shown
here

stiffener

$BSF := 3.94 \quad SDEPTH := 5. \quad TSF := .215 \quad TSW := .17$

plate

try this with SDEPTH = 6, to see yield effect

$a := 8 \cdot 12 \quad b := 25 \quad t := .5 \quad N := 2$

$L := a \quad L = 96$

material

$\sigma_Y := 80 \cdot 10^3 \quad \upsilon := 0.3 \quad E := 29.6 \cdot 10^6 \quad D := \dfrac{E \cdot t^3}{12 \cdot \left(1 - \upsilon^2\right)}$

general parameters:

$HSW := SDEPTH - TSF \qquad A_w := (SDEPTH - TSF) \cdot TSW \qquad A_f := BSF \cdot TSF \qquad A_s := A_w + A_f$

$HSW = 4.79 \qquad A_w = 0.81 \qquad A_f = 0.85 \qquad A_s = 1.66$

$d := SDEPTH - \dfrac{TSF}{2} + \dfrac{t}{2} \qquad B := (N + 1) \cdot b \qquad A_p := b \cdot t \qquad \dfrac{b \cdot t}{A_s} = 7.53 \qquad \delta_x := \dfrac{A_s}{b \cdot t}$

$d = 5.14 \qquad B = 75 \qquad A_p = 12.5 \qquad \delta_x = 0.13$

parameters that are functions of b_e

let: $b_e := 0.7657 \cdot b$ as a place to start and for printing purposes. Can also use be directly and compare with $b_r(b_e)$. If $b_r(b_e)$/ b < start use result until converges.

$$A_e(b_e) := A_s + b_e \cdot t \qquad C_1(b_e) := \frac{A_w \cdot \left(\dfrac{A_e(b_e)}{3} - \dfrac{A_w}{4}\right) + A_f \cdot b_e \cdot t}{\left(A_e(b_e)\right)^2} \qquad I_e(b_e) := A_e(b_e) \cdot (d)^2 \cdot C_1(b_e)$$

$A_e(b_e) = 11.23 \qquad C_1(b_e) = 0.09 \qquad I_e(b_e) = 25.87$

$$\rho_e(b_e) := \sqrt{\frac{I_e(b_e)}{A_e(b_e)}} \qquad \gamma_x(b_e) := \frac{12 \cdot \left(1 - \upsilon^2\right) \cdot I_e(b_e)}{b_e \cdot (t)^3} \qquad C_\pi := \min\left[\left[\begin{matrix} \dfrac{B}{a} \cdot \sqrt{\dfrac{\gamma_x(b_e)}{2 \cdot \left(1 + \sqrt{1 + \gamma_x(b_e)}\right)}} \\ 1 \end{matrix}\right]\right]$$

$\rho_e(b_e) = 1.52 \qquad \gamma_x(b_e) = 118.07$

for iteration

$C_\pi = 1$

$$b_r(b_e) := \frac{C_\pi \cdot a \cdot t}{\rho_e(b_e) \cdot \sqrt{3 \cdot \left(1 - \upsilon^2\right)}} \qquad \frac{b_r(b_e)}{b} = 0.7657 \qquad \text{Checks??} \qquad b_{rat} := \frac{b_r(b_e)}{b} \qquad b_{rat} = 0.766$$

$b_r(b_e) = 19.14$

critical stress relationships:

stiffener & plate as a column

$$\sigma_{ecr}(b_e) := \frac{-\pi^2 \cdot E}{\left(\frac{C_\pi \cdot L}{\rho_e(b_e)}\right)^2}$$

$$\sigma_{ecr}(b_e) = -73015$$

effective portion of plate

$$\sigma_{ecr\_pl}(b_e) := -4 \cdot \frac{\pi^2 \cdot D}{b_e^2 \cdot t}$$

$$\sigma_{ecr\_pl}(b_e) = -73008$$

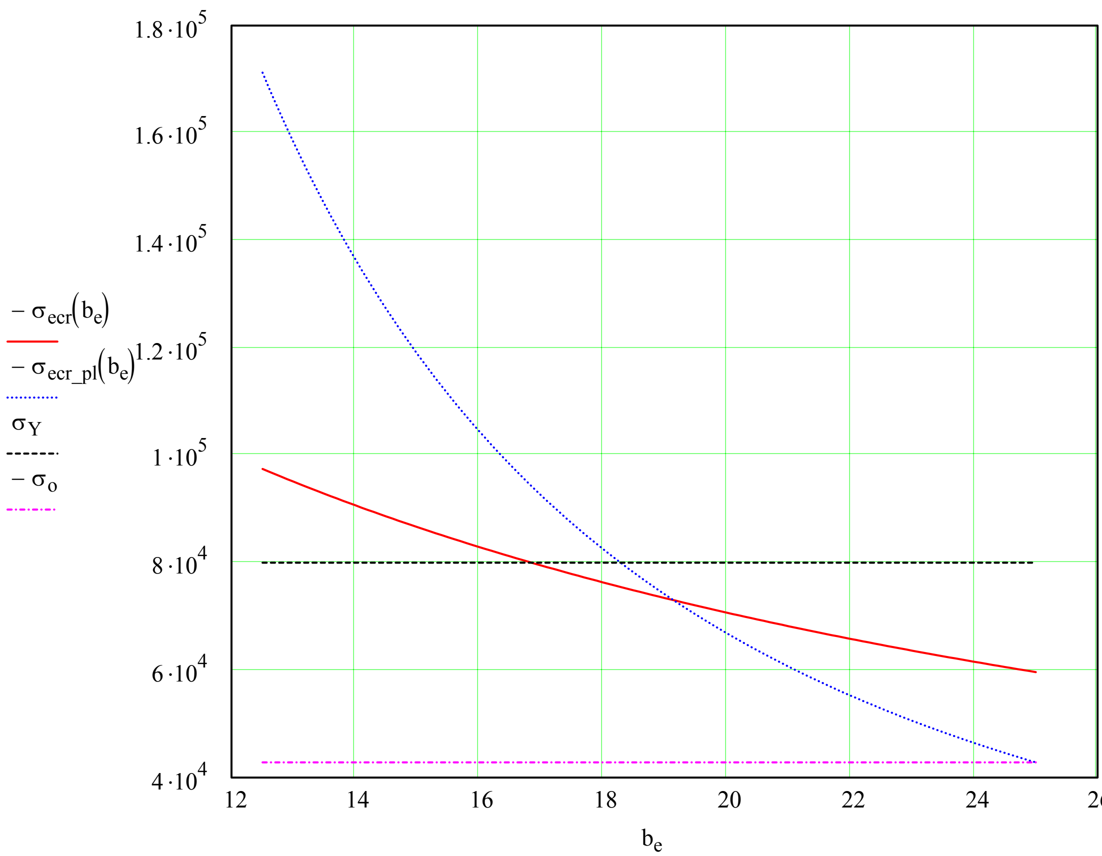
original plate

$$\sigma_o := 3.62 \cdot E \cdot \left(\frac{t}{b}\right)^2$$ OR.... $$\sigma_o := -4 \cdot \frac{\pi^2 \cdot D}{b^2 \cdot t}$$

$\sigma_o = -42804$ $\sigma_o = -42804$

for plotting to determine intersection;

$$b_e := \frac{b}{2}, \frac{b}{2} + 0.1 .. b$$



now obseving intersection: $b_e := b_{rat} \cdot b$ $b_e = 19.14$

assuming iteration complete and matched above.

stiffener & plate as a column

$$\sigma_{ecr}(b_e) := \frac{-\pi^2 \cdot E}{\left(\frac{C_\pi \cdot L}{\rho_e(b_e)}\right)^2}$$

$$\sigma_{ecr}(b_e) = -73018$$

effective portion of plate

$$\sigma_{ecr\_pl}(b_e) := -4 \cdot \frac{\pi^2 \cdot D}{b_e^2 \cdot t}$$

$$\sigma_{ecr\_pl}(b_e) = -73015$$

translation back to applied stress

$$\sigma_{axcr} := \left(\frac{b_e \cdot t + A_s}{b \cdot t + A_s}\right) \cdot \sigma_{ecr}(b_e)$$

$$\sigma_{axcr} = -57913$$

partial safety factor

$\gamma_C := 1.5$ $\sigma_C := -20000$ input

$$\gamma R_{PCCB} := \gamma_C \cdot \frac{\sigma_C}{\sigma_{axcr}}$$

$$\gamma R_{PCCB} = 0.51801$$

is it slender?? λ>1 ??

$$\lambda := \frac{a}{\pi \cdot \rho_e(b_e)} \cdot \sqrt{\frac{\sigma_Y}{E}}$$

$$\lambda = 1.05$$