Massachusetts Institute of Technology
Department of Mechanical Engineering

2.12 Introduction to Robotics

Problem Set No.5

Out: October 19, 2005 Due: October 26, 2005

Problem 1

Consider a mass-less rod of length *l* constrained by two sliding joints at both ends A and B, as shown in the figure below. The rod is connected to a spring of spring constant *k* at A and is pulled down by mass *m* at B. Friction is negligible. Let θ be the angle between the horizontal line and the rod. Using the *Principle of Virtual Work*, show that the rod is in equilibrium at the angle θ that satisfies the following relationship:

$$(1-\cos\theta)\tan\theta = \frac{mg}{k\ell}$$

where *g* is acceleration of gravity. Assume that at $\theta = 0$ the spring force is zero.

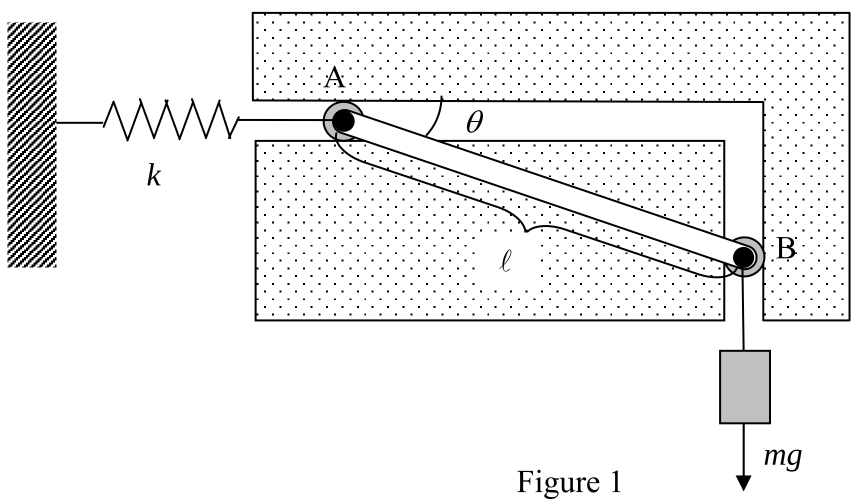

Figure 1

Problem 2

A planar robot with three revolute joints is shown below. Let θ_i and ℓ_i be the angle of joint *i* and the length of link *i* , respectively, and x_e, y_e, ϕ_e be the end-effecter position and orientation viewed from the base coordinate frame, as shown in the figure.

In performing a class of tasks, the end-effecter orientation doesn't have to be specified. Namely, the number of controlled variables is two, while the number of degrees of freedom is three. Therefore the robot has a redundant degree of freedom.

At an arm configuration of $\theta_1 = 135^o$, $\theta_2 = 45^o$, $\theta_3 = 225^o$ obtain the 2x3 Jacobian matrix relating the end-effecter position to joint displacements. We want to generate an endpoint force of $F_x = 10\,N$, $F_y = -2\,N$. Obtain the equivalent joint torques needed for generating the endpoint force.

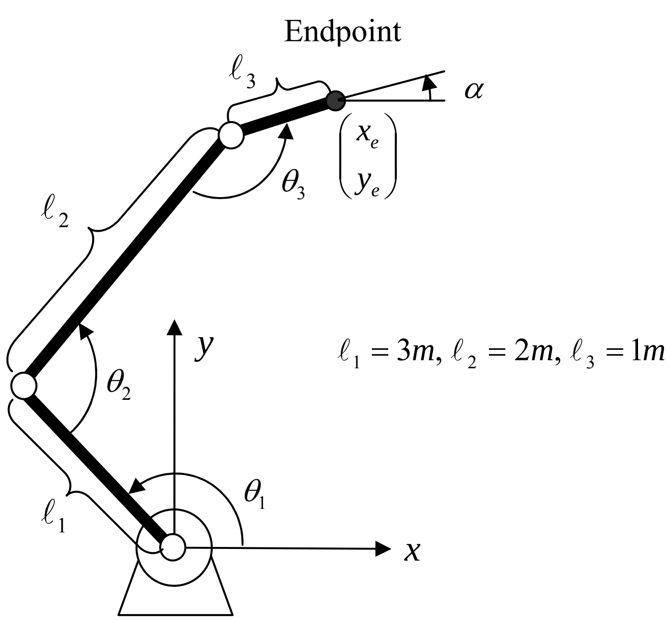

Figure 2 Three degree-of-freedom redundant robot arm

Note: In the following problem, numerical values of link lengths and other geometric parameters are not given, but you can solve the problem using given functions $h_1(s_1)$, $h_2(s_1, s_2)$ alone.

Problem 3

'Text and diagram removed for copyright reasons. See Problem 4.2, description and figure, in Asada and Slotine, 1986.'

(1) At a given configuration of Θ_1 and Θ_2 we want to move the endpoint at a specified velocity, $v = [v_x, v_y]^T$ with reference to the base coordinate system O_O - xy. Obtain the cylinder speeds, s_1 and s_2, that produce the desired endpoint velocity. Hint: Use derivatives of functions $h_1 (s_1)$ and $h_2 (s_{1,} s_2)$.

(2) Let f_1 and f_2 be the forces exerted by the cylinders, HC_1 and $HC_{2,}$ respectively. Each force acts in the longitudinal direction of the cylinder, and is defined to be positive in the direction of expanding the cylinder. We want to push an object at the arm's endpoint. Obtain the cylinder forces, f_1 and f_2, required for exerting an endpoint force of $F_x = 0$ and $F_y = F$, assuming that all the joints are frictionless. Also ignore gravity.