2.14/2.140 Problem Set 11

Assigned: Wed. Apr. 23, 2014
Due: Wed. April 30, 2014, in class
Reading: FPE Sections 8.1–8.4

Problem 1: FPE 8.7 For this problem, in part a), calculate by hand the requested magnitude expression $|H(z_1)|$ where $z_1 = e^{j\omega_1 T}$, and $\omega_1 = 3$ rad/sec. (The problem statement omits the subscript 1 in the first sentence.) In part b), you may use Matlab to generate the requested Bode plots. How do these compare to what you would expect from the continuous-time system?

Problem 2: This problem considers designing a continuous-time controller for a single-axis magnetic levitation device. This controller is then mapped into an approximating discrete-time controller.

The plant to be controlled is a mass driven by a voice-coil actuator. Assume that this plant has a transfer function

$$G_p(s) = 1000/s^2$$

a) For this plant, design a continuous-time PID controller using the series-lag-lead form

$$G_c(s) = K_p \left(1 + \frac{1}{T_I s}\right)\left(\frac{\alpha\tau s + 1}{\tau s + 1}\right)$$

to achieve a crossover frequency of $\omega_c = 1000$ rad/sec, with a phase margin of $\phi_m = 45°$. Your continuous-time design is to meet specifications for the specified continuous time plant, with no consideration of the later sampling operation when implemented as a discrete-time approximation. You should be able to accomplish this design using hand-calculations and Bode plot sketches. Clearly show your design effort. Indicate the loop crossover frequency, phase margin and gain margin.

b) Now map this controller to discrete-time using the Tustin transformation, and assuming a sample rate of 6 kHz. We recommend using the operator notation introduced in class to aid in this step. Show your hand calculations that give the discrete-time controller. We recommend that you discretize the three terms of the PID controller separately: 1) gain, 2) lag, 3) lead. This will be easier than discretizing the full second-order controller. Give a block diagram for the discrete-time controller showing the transfer functions in this block diagram. What are the resulting difference equations for each element? Carefully show your design approach and calculations. (For the purposes of this problem, do the approximations by hand, rather than directly using Matlab to generate the transformations.)

c) Use Matlab/Simulink to simulate the step response of your closed-loop system with i) the continuous-time PID controller, and ii) the approximating discrete-time controller which you designed. Please include plots of the control signal (plant input) and the plant output for each case. How do these responses compare?

Note that you will need to choose the integration solver routines and sample times in Simulink to properly simulate the continuous-time loop. That is, don't use a fixed-step solver with 6 kHz in Simulink when you want to simulate the continuous-time loop. If these considerations are unfamiliar, please consult any of the online Simulink tutorials to understand the solvers and time-steps used to simulate continous-time and mixed discrete-time/continuous-time systems.

In your Simulink simulations, reduce the discrete-time sample rate from 6 kHz, and comment on the effect of longer sample times in the resulting signals. Include relevant plots to show these effects. At about what reduced sample rate does the system go unstable?

Recognize that you can implement the Simlink model block diagram in the series lag lead form, replacing each block with its discrete equivalent.

When we ask you to simulate the continuous-time loop, this should be done with the continuous-time controller. When we ask you to simulate the discrete-time loop, use discrete-time transfer functions in Simulink, and input the difference equation parameters that you calculate when discretizing.